EE 330
Lecture 17

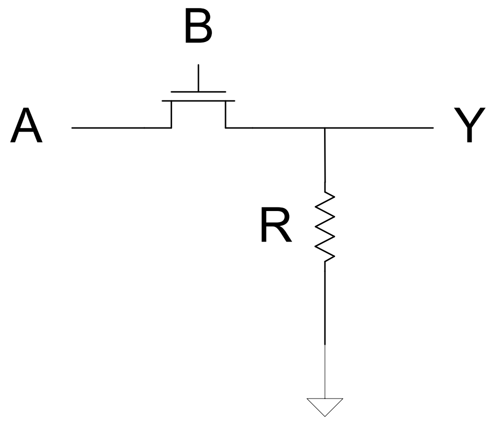
MOSFET Modeling

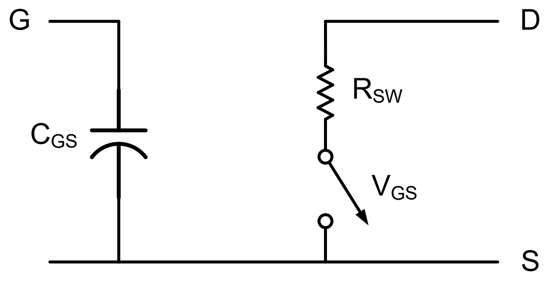
Fall 2024 Exam Schedule

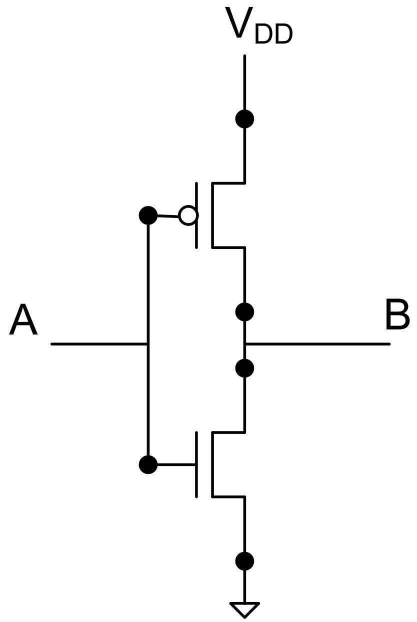
Exam 1	Friday	Sept 27
Exam 2	Friday	October 25
Exam 3	Friday	Nov 22
Final Exam	Monday	Dec 16 12:00 - 2:00 PM

Review from last lecture

Limitations of Existing MOSFET Models







For minimum-sized devices in a 0.5u process with V_{DD}=5V

$$C_{GS} \cong 1.5fF$$

$$R_{sw} \cong \left.\begin{matrix} 2K\Omega \;\; n-channel \\ 6K\Omega \;\; p-channel \end{matrix}\right\}$$

What is Y when A=B=V_{DD}

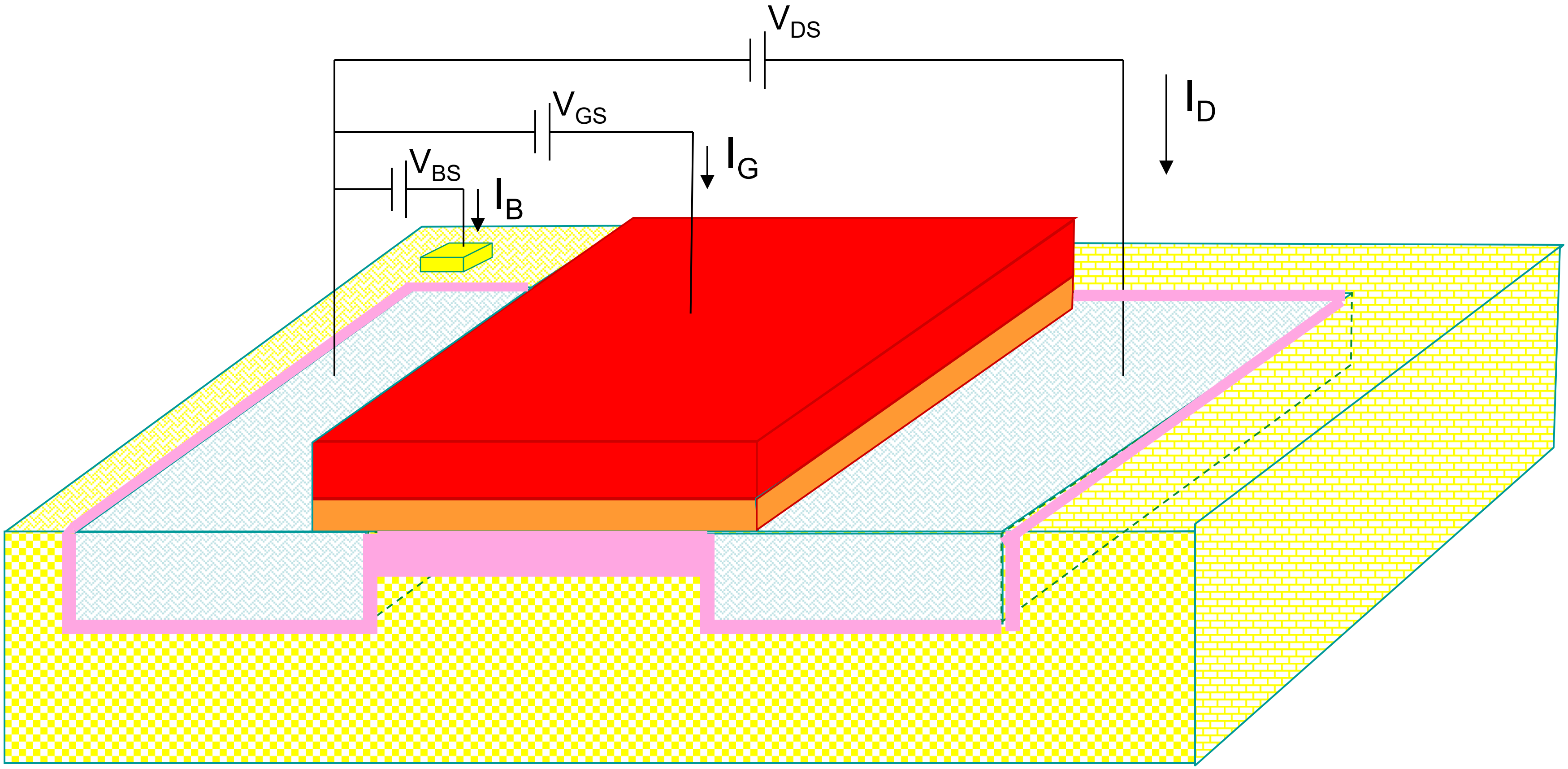
What is R_{SW} if MOSFET is not minimum sized?

What is power dissipation if A is stuck at an intermediate voltage?

Better Model of MOSFET is Needed!

Review from last lecture

n-Channel MOSFET Operation and Model



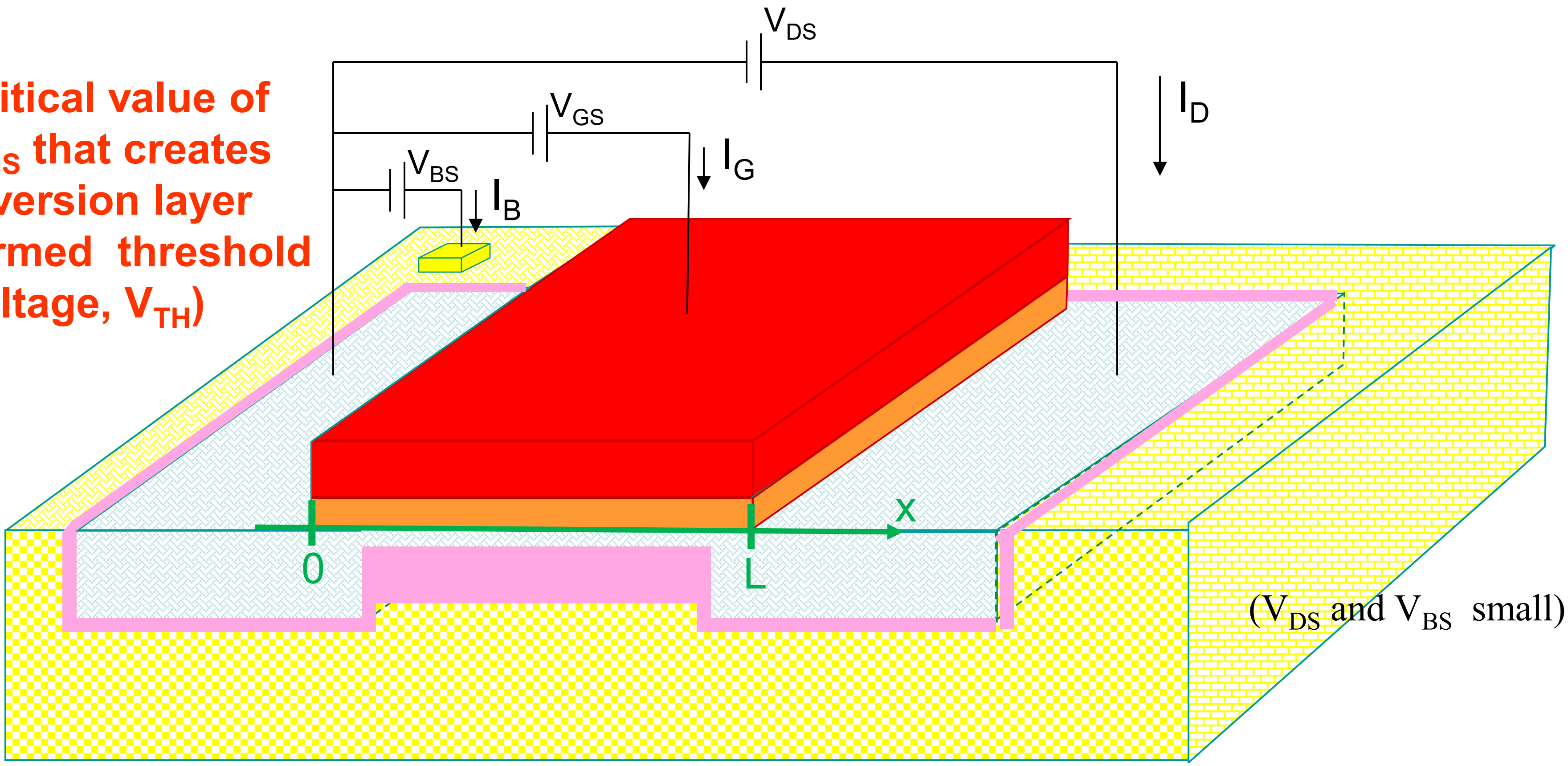
$I_D=0$
$I_G=0$
$I_B=0$

Model in Cutoff Region

Review from last lecture

n-Channel MOSFET Operation and Model

Critical value of V_{GS} that creates inversion layer termed threshold voltage, V_{TH})



(V_{DS} and V_{BS} small)

Increase V_{GS} more

Inversion layer forms in channel

Inversion layer will support current flow from D to S

Channel behaves as thin-film resistor

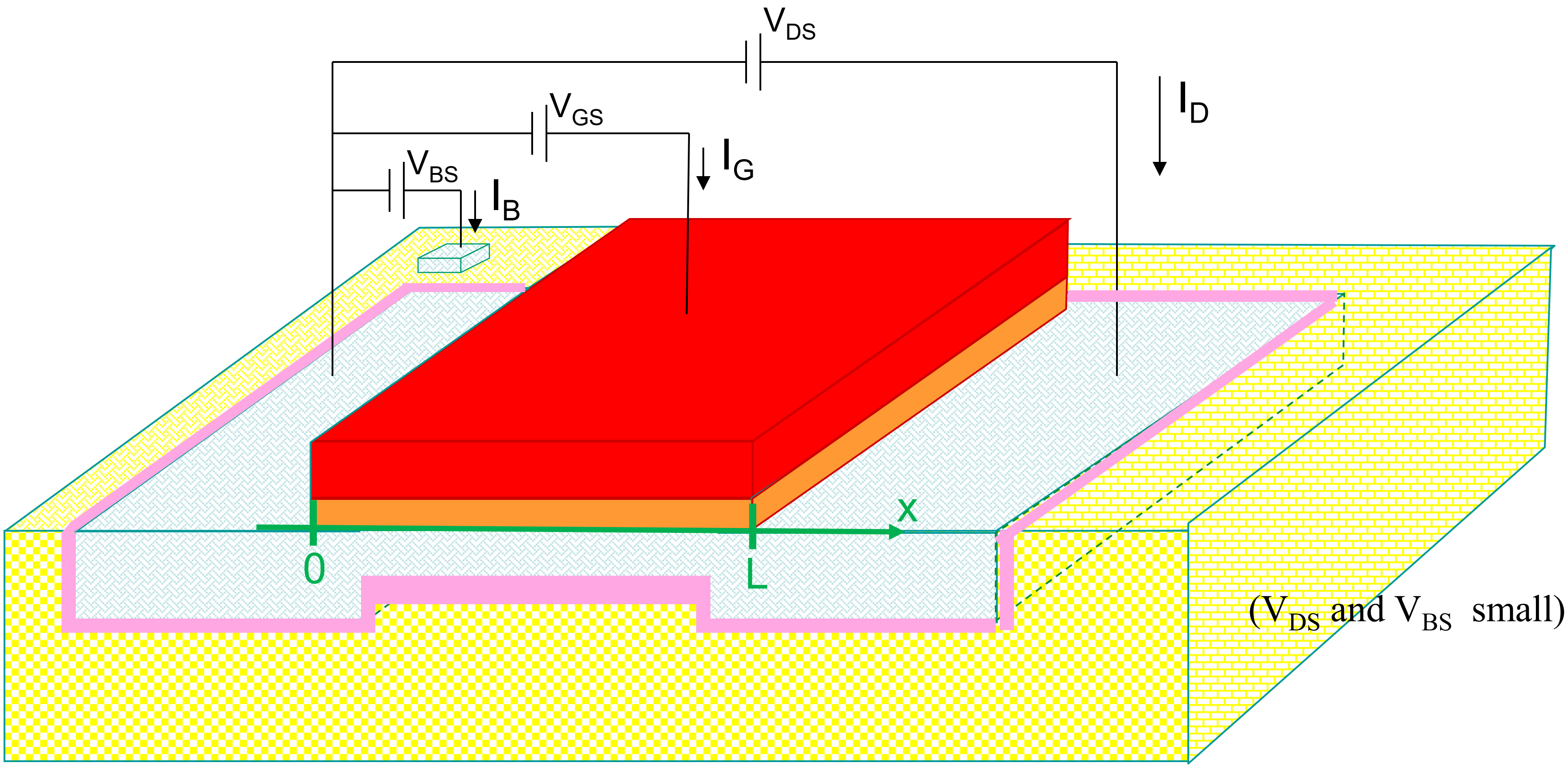
$I_D R_{CH} = V_{DS}$

$I_G = 0$

$I_B = 0$

Review from last lecture

n-Channel MOSFET Operation and Model



$V_{GC}(x)$ approx. constant for small V_{DS}

Increase V_{GS} more (with V_{DS} and V_{BS} still small)

Inversion layer in channel thickens

R_{CH} will decrease

Termed “ohmic” or “triode” region of operation

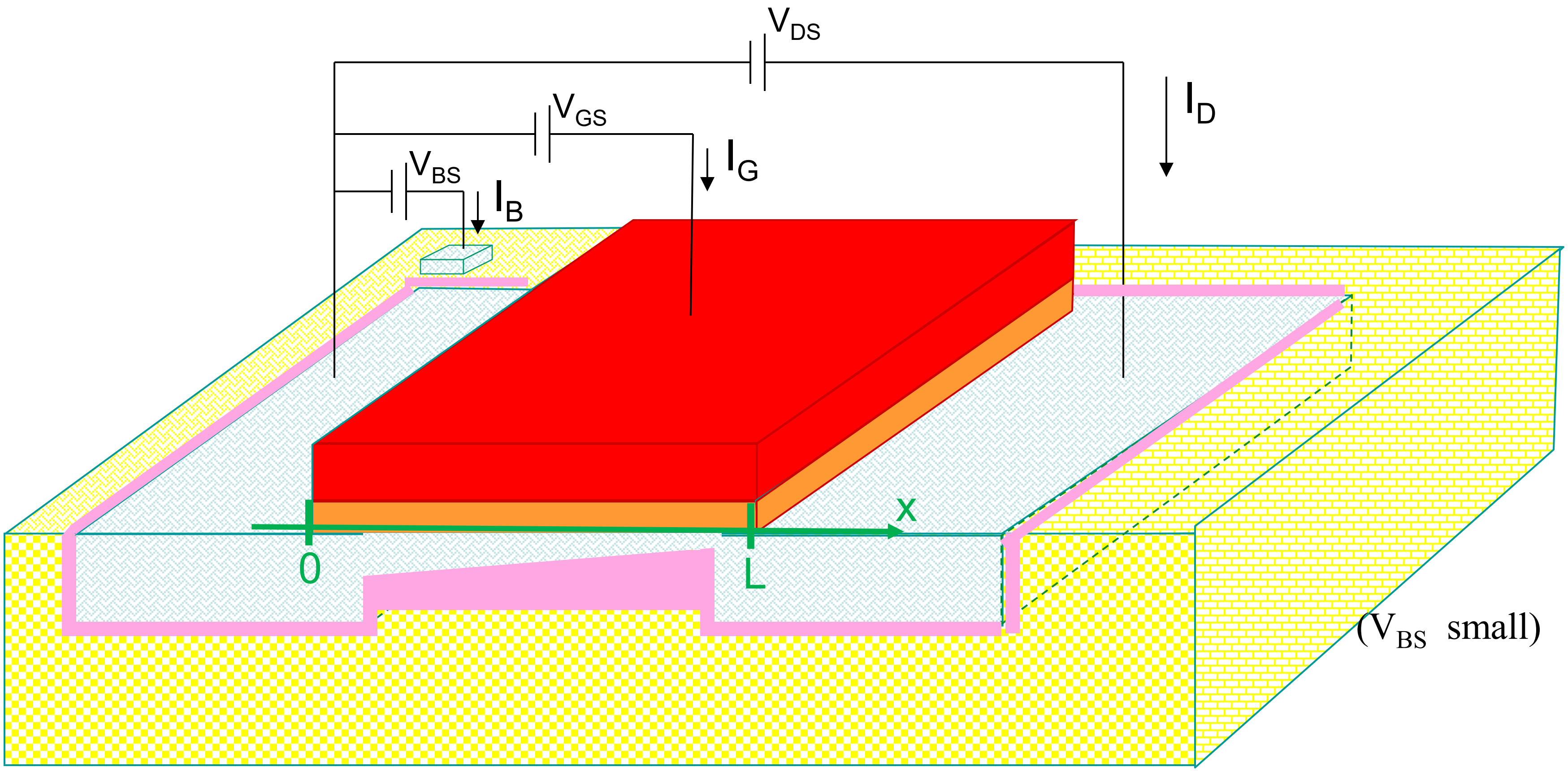
$I_D R_{CH} = V_{DS}$

$I_G = 0$

$I_B = 0$

Review from last lecture

n-Channel MOSFET Operation and Model



$V_{GC}(x)$ changes with x for larger V_{DS}

Increase V_{DS}

Inversion layer thins near drain

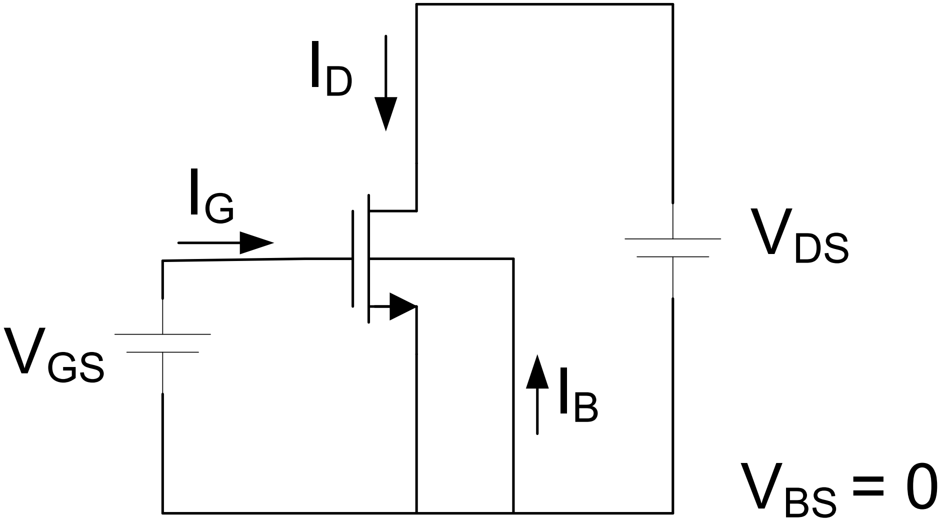
I_D no longer linearly dependent upon V_{DS}

Still termed “ohmic” or “triode” region of operation

$I_D=?$
$I_G=0$
$I_B=0$

Review from last lecture

Triode Region of Operation



For V_{DS} larger

$$\xcancel{R_{CH}=\frac{L}{W}\frac{1}{(V_{GS}-V_{TH})\mu C_{OX}}}$$

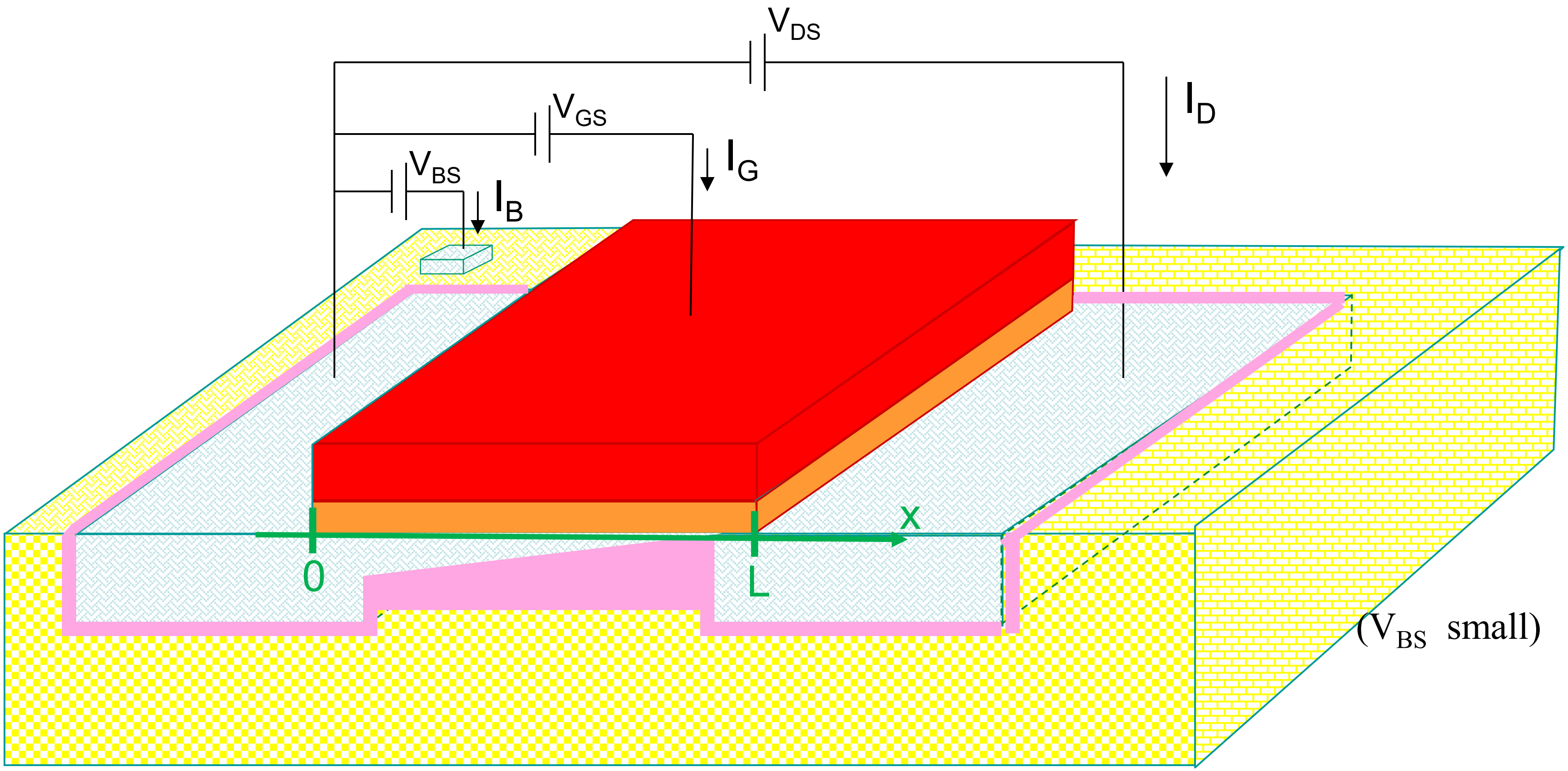
$$I_D=\mu C_{OX}\frac{W}{L}\left(V_{GS}-V_{TH}-\frac{V_{DS}}{2}\right)V_{DS}$$

$$I_G=I_B=0$$

Model in Triode Region

Review from last lecture

n-Channel MOSFET Operation and Model



Increase V_{DS} even more

$V_{GC}(L) = V_{TH}$ when channel saturates

Inversion layer disappears near drain

Termed “saturation”region of operation

Saturation first occurs when $V_{DS}=V_{GS}-V_{TH}$

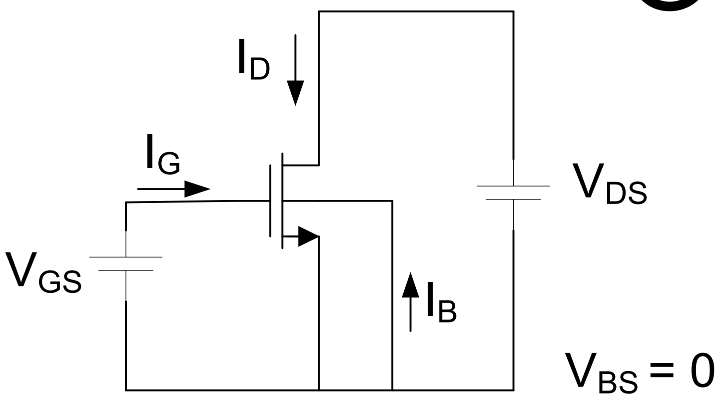
$I_D=?$

$I_G=0$

$I_B=0$

Review from last lecture

Saturation Region of Operation



$$I_D = \mu C_{OX}\frac{W}{L}\left(V_{GS} - V_{TH} - \frac{V_{DS}}{2}\right)V_{DS}$$

or equivalently

For V_{DS} at onset of saturation ______________

$$I_D = \mu C_{OX}\frac{W}{L}\left(V_{GS} - V_{TH} - \frac{V_{GS} - V_{TH}}{2}\right)\left(V_{GS} - V_{TH}\right)$$

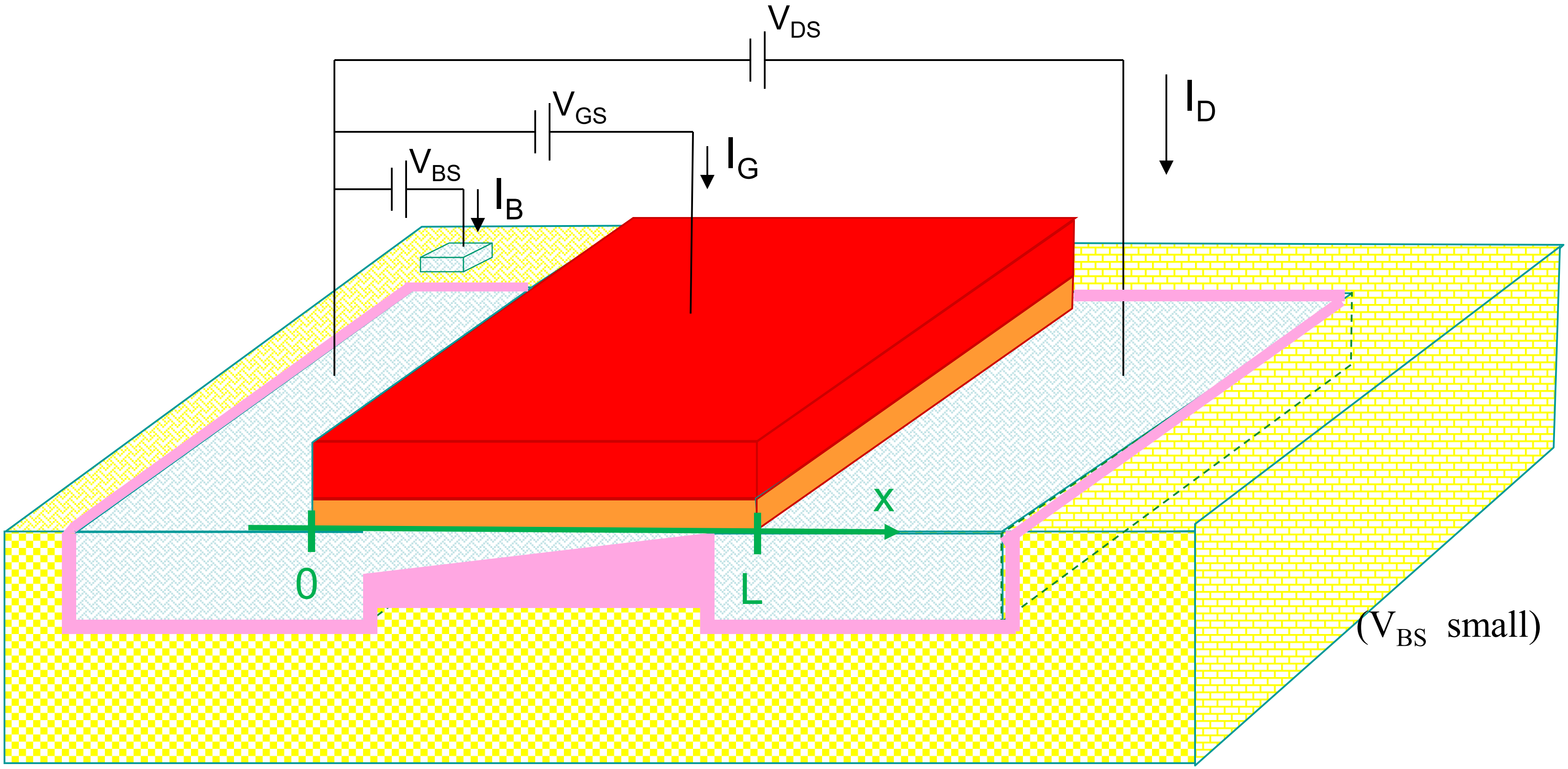
or equivalently

$$I_D = \frac{\mu C_{OX} W}{2L}\left(V_{GS} - V_{TH}\right)^2$$

$$I_G = I_B = 0$$

Review from last lecture

n-Channel MOSFET Operation and Model



Increase V_{DS} even more (beyond V_{GS}-V_{TH})

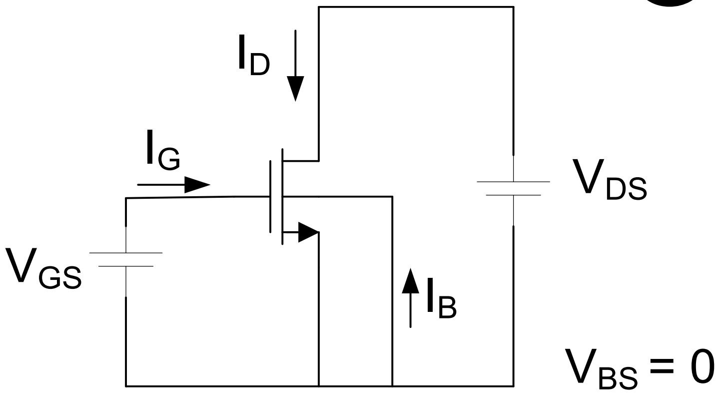
Nothing much changes !!

Termed “saturation”region of operation

I_D=?
I_G=0
I_B=0

Review from last lecture

Saturation Region of Operation



For V_{DS} in Saturation

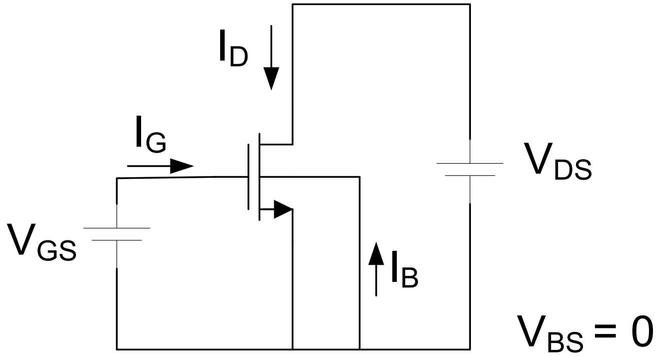
$$\boxed{\begin{array}{l} I_D = \frac{\mu C_{OX} W}{2L}\left(V_{GS} - V_{TH}\right)^2 \\ I_G = I_B = 0 \end{array}}$$

Model in Saturation Region

Review from last lecture

Model Summary

n-channel MOSFET



Notation change: $V_T = V_{TH}$, don't confuse V_T with $V_t = kT/q$

$$I_D = \begin{cases} 0 & V_{GS} \le V_{TH} & \text{Cutoff} \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_{TH} - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_{TH} \quad V_{DS} < V_{GS} - V_{TH} & \text{Triode} \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_{TH}\right)^2 & V_{GS} \ge V_{TH} \quad V_{DS} \ge V_{GS} - V_{TH} & \text{Saturation} \end{cases}$$

$$I_G = I_B = 0$$

Model Parameters: {μ, V_{TH}, C_{OX}} Design Parameters : {W, L}

This is a piecewise model (not piecewise linear though)

Piecewise model is continuous at transition between regions

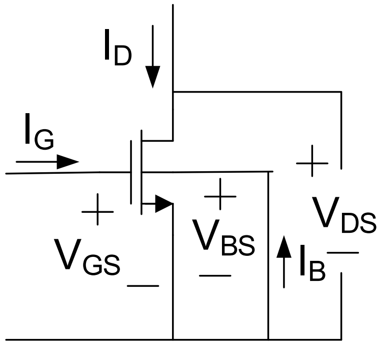
(Deep triode special case of triode where V_{DS} is small $R_{CH} = \dfrac{L}{W}\dfrac{1}{\left(V_{GS} - V_{TH}\right)\mu C_{OX}}$)

Note: This is the third model we have introduced for the MOSFET

Review from last lecture

Model Summary

n-channel MOSFET



$V_{BS} = 0$

$$I_D = \begin{cases} 0 & V_{GS} \le V_{TH} \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_{TH} - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_{TH} \quad V_{DS} < V_{GS} - V_{TH} \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_{TH}\right)^2 & V_{GS} \ge V_{TH} \quad V_{DS} \ge V_{GS} - V_{TH} \end{cases}$$

$$I_G = I_B = 0$$

Observations about this model (developed for V_{BS}=0):

$$I_D = f_1\left(V_{GS}, V_{DS}\right)$$

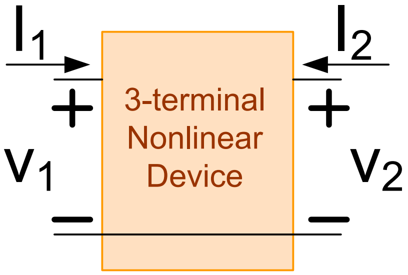
$$I_G = f_2\left(V_{GS}, V_{DS}\right)$$

$$I_B = f_3\left(V_{GS}, V_{DS}\right)$$

This is a nonlinear model characterized by the functions f_1, f_2, and f_3 where we have assumed that the port voltages V_{GS} and V_{DS} are the independent variables and the drain currents are the dependent variables

Review from last lecture

General Nonlinear Models

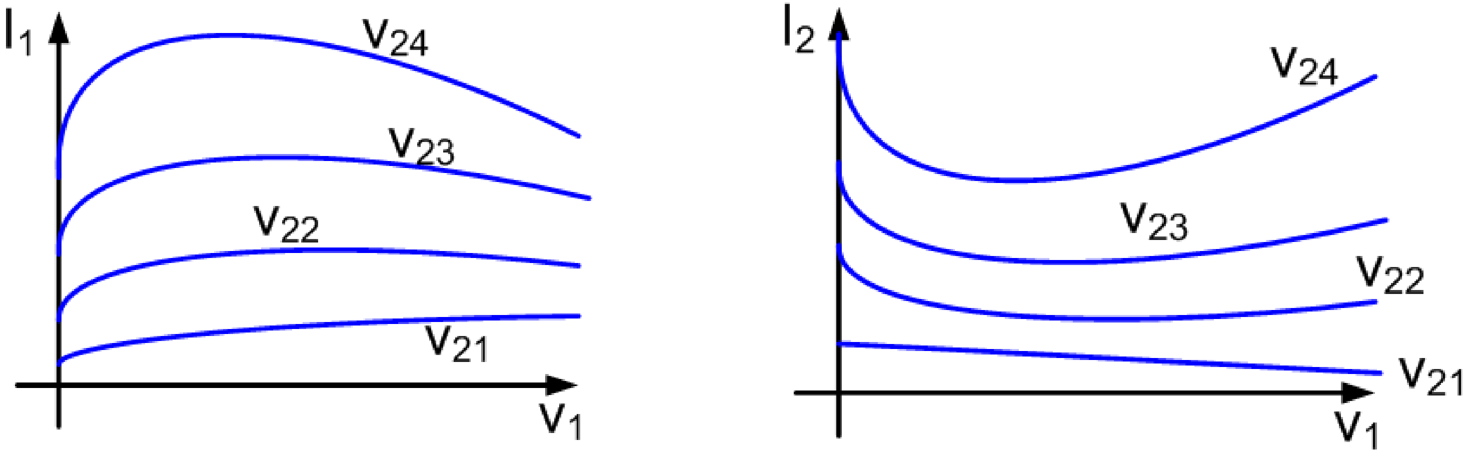


$$I_1 = f_1(V_1, V_2)$$

$$I_2 = f_2(V_1, V_2)$$

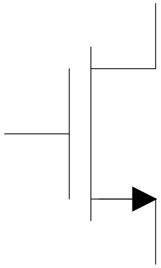
I_1 and I_2 are 3-dimensional relationships which are often difficult to visualize

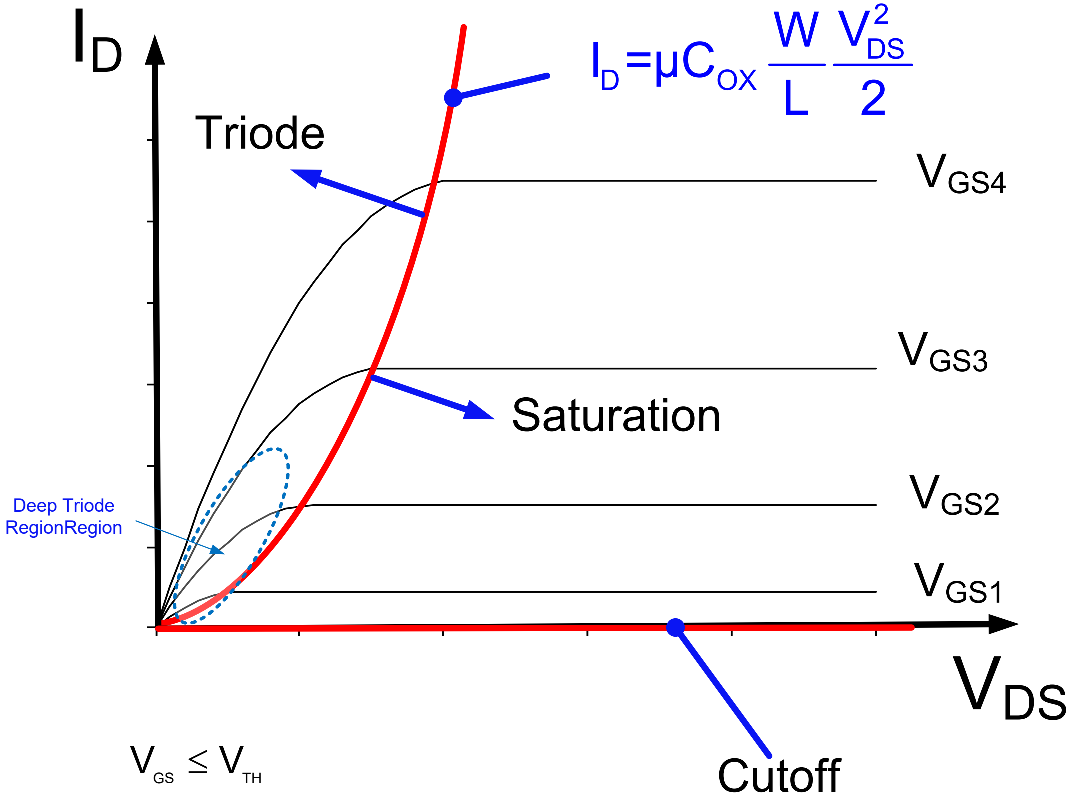
Two-dimensional representation of 3-dimensional relationships



Review from last lecture

Graphical Representation of MOS Model





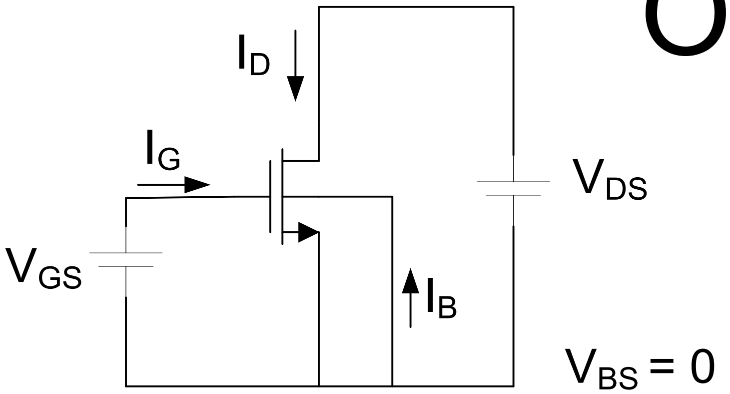
$$I_D = \begin{cases} 0 & V_{GS} \le V_{TH} \\ \mu C_{OX} \frac{W}{L}\left(V_{GS} - V_{TH} - \frac{V_{DS}}{2}\right) V_{DS} & V_{GS} \ge V_{TH} \quad V_{DS} < V_{GS} - V_{TH} \\ \mu C_{OX} \frac{W}{2L}\left(V_{GS} - V_{TH}\right)^2 & V_{GS} \ge V_{TH} \quad V_{DS} \ge V_{GS} - V_{TH} \end{cases}$$

$$I_G = I_B = 0$$

Parabola separated triode and saturation regions and corresponds to $V_{DS}=V_{GS}-V_{TH}$

Review from last lecture

Saturation Region of Operation

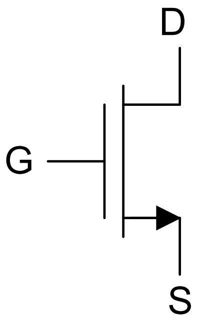


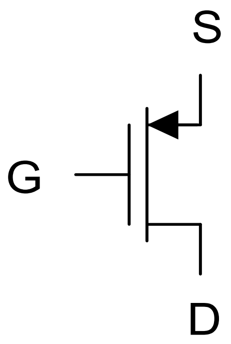
For V_{DS} in Saturation

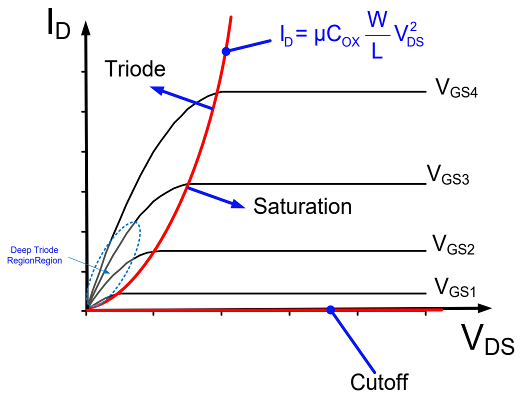
$$\boxed{\begin{aligned} I_D &= \frac{\mu C_{OX} W}{2L}\left(V_{GS} - V_{TH}\right)^2 \\ I_G &= I_B = 0 \end{aligned}}$$

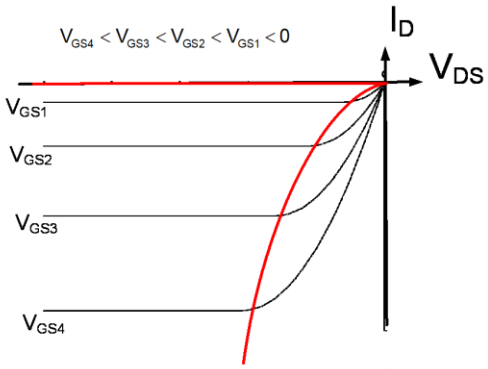
Model in Saturation Region

PMOS and NMOS Models



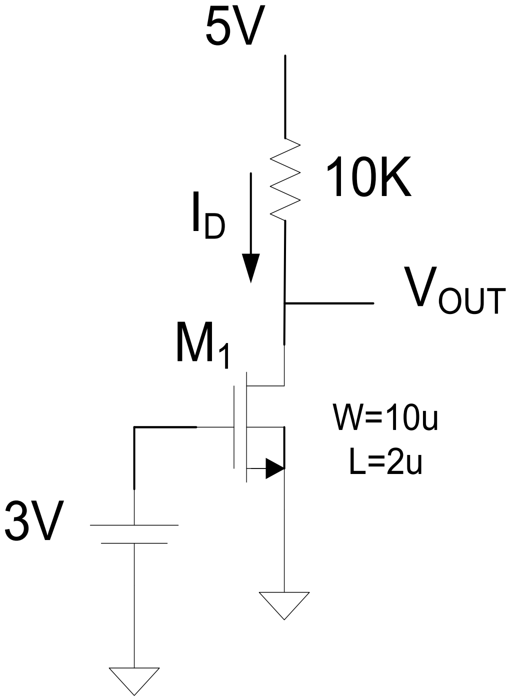






- Functional form identical, sign changes and parameter values different
- Will give details about p-channel model later

Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume V_{TH}=1V and μC_{OX}=100μAV^{-2}

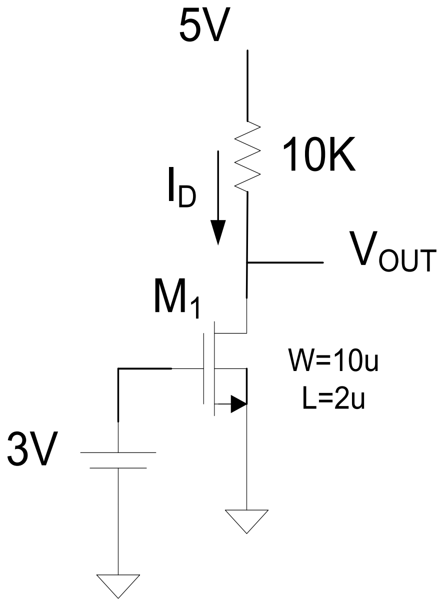


Solution:

Since $V_{GS}>V_{TH}$, M_1 is operating in either saturation or triode region

Strategy will be to guess region of operation, solve, and then verify region

Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume V_{TH}=1V and μC_{OX}=100μAV^{-2}



Solution:

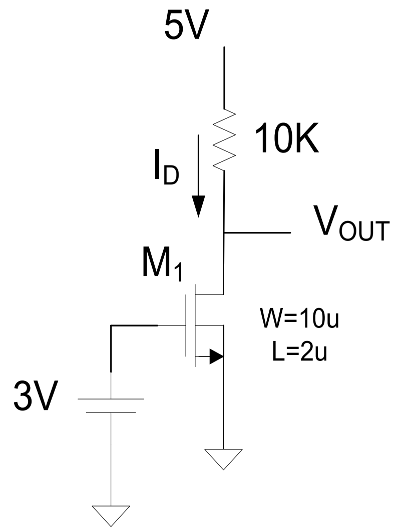
Guess M_1 in saturation

$$\left.\begin{array}{l} 5V = I_D 10K + V_{OUT} \\ I_D = \dfrac{\mu C_{OX} W}{2L}(3-V_{TH})^2 \end{array}\right\}$$

Required verification: $V_{DS} > V_{GS} - V_{TH}$

Can eliminate I_D between these 2 equations to obtain V_{OUT}

Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume V_{TH}=1V and $\mu C_{OX}=100\mu AV^{-2}$



Guess M_1 in saturation

Required verification: $V_{DS}>V_{GS}-V_T$

$$\left.\begin{aligned} 5V &= I_D 10K + V_{OUT} \\ I_D &= \frac{\mu C_{OX} W}{2L}(3-V_{TH})^2 \end{aligned}\right\}$$

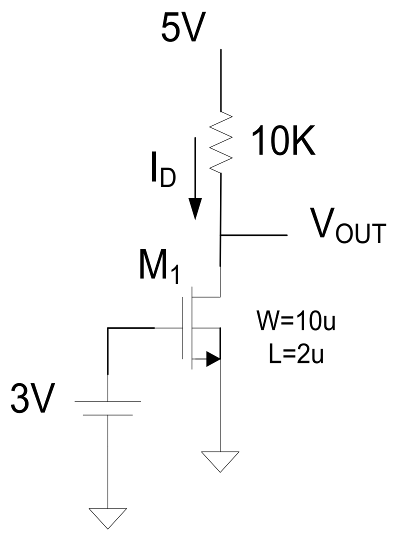
$$V_{OUT} = 5V-10K\left[\frac{100\mu AV^{-2}10\mu}{2\bullet 2\mu}(2V)^2\right]$$

$$V_{OUT} = -5V$$

Verification: $V_{DS}=V_{OUT}$

-5 >? 2V - - 0 No! So verification fails and Guess of region is invalid

Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume V_{TH}=1V and $\mu C_{OX}=100\mu AV^{-2}$



Guess M_1 in triode

Required verification: $V_{DS}<V_{GS}-V_T$

$$\left.\begin{array}{l} 5V=I_D 10K+V_{OUT} \\ I_D=\dfrac{\mu C_{OX}W}{L}\left(3-V_{TH}-\dfrac{V_{DS}}{2}\right)V_{DS}\end{array}\right\}$$

$$V_{OUT}=5V-10K\left[\frac{100\mu AV^{-2}10\mu}{2\mu}\left(2V-\frac{V_{OUT}}{2}\right)V_{OUT}\right]$$

$$V_{OUT}=5V-\left[5\left(2V-\frac{V_{OUT}}{2}\right)V_{OUT}\right]$$

Solving for V_{OUT}, obtain

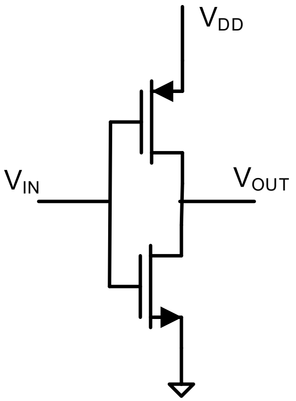
$$V_{OUT}=0.515V$$

Verification: $V_{DS}=V_{OUT}$

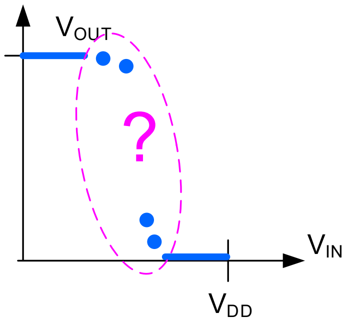
0.515 <? 2V Yes! So verification succeeds and triode region is valid

$$\boxed{V_{OUT}=0.515V}$$

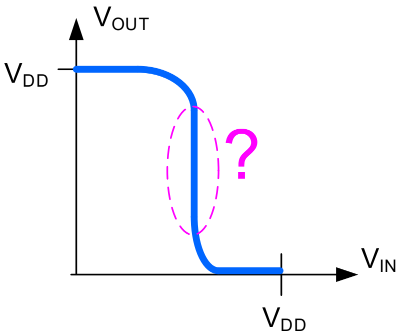
Limitations of Existing Models



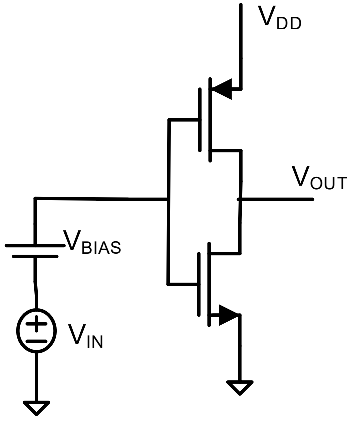
Logic Gate



Switch-Level Models



Simple square-law Model



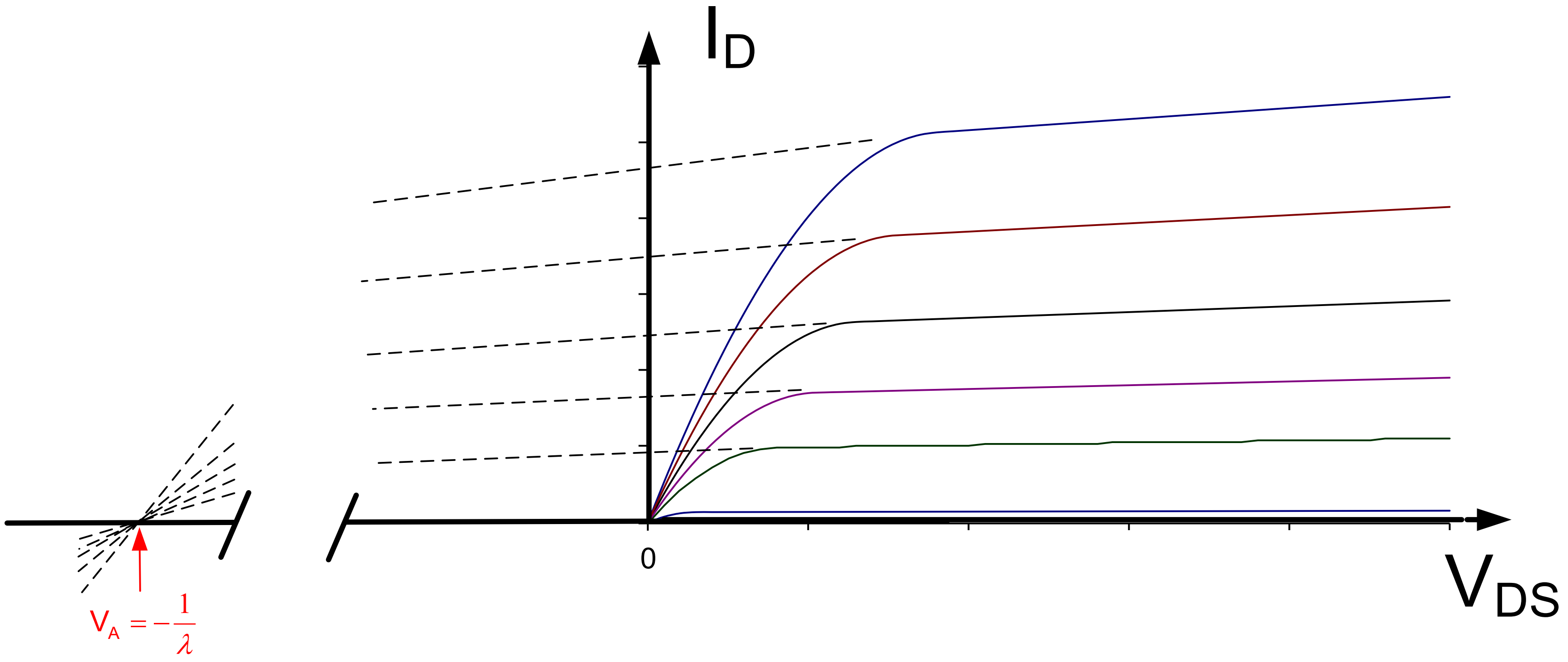
Voltage Amplifier

Switch-Level Models

Simple square-law Model }

Voltage Gain
Input/Output Relationship ?

Model Extensions

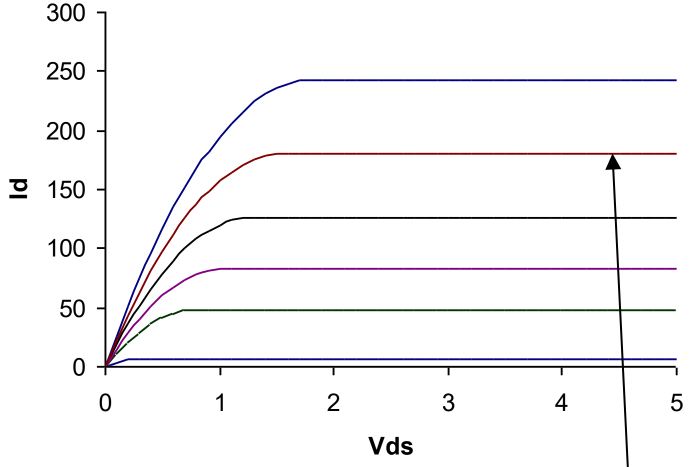


Projections intersect $-V_{DS}$ axis at same point, termed Early Voltage

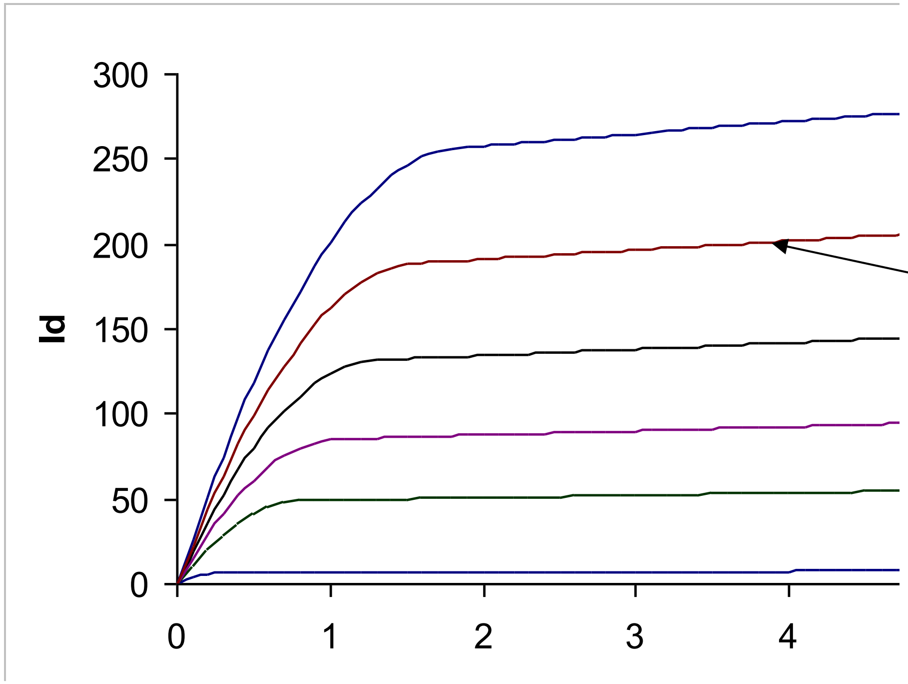
Typical values from -20V to -200V

Usually use parameter λ instead of V_A in MOS model

Model Extensions



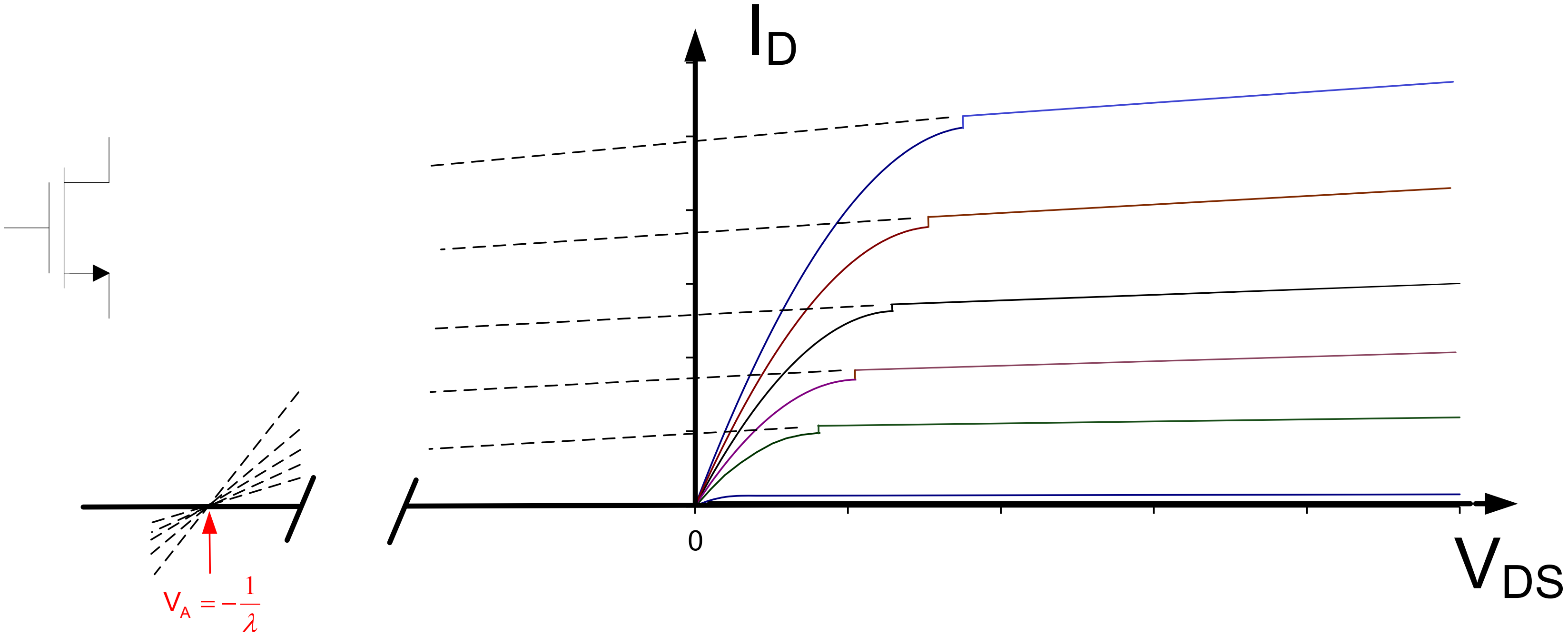
Existing Model



Actual Device

Slope is not 0

Model Extensions



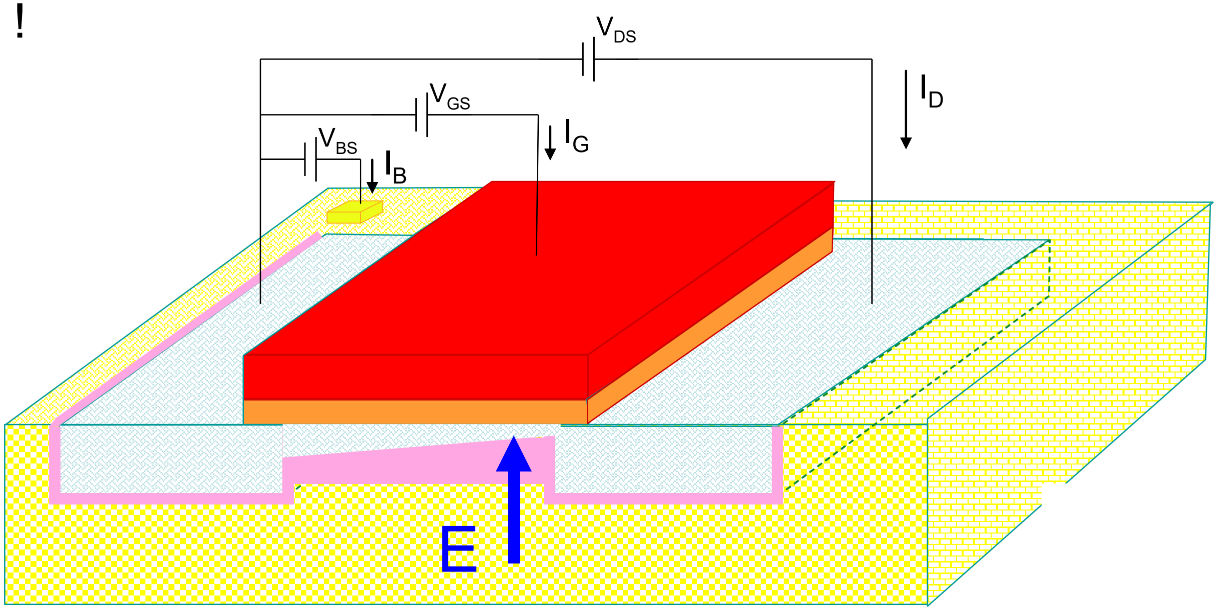
$$I_D = \begin{cases} 0 & V_{GS} \le V_{TH} \\ \mu C_{OX} \frac{W}{L}\left(V_{GS} - V_{TH} - \frac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_{TH} \quad V_{DS} < V_{GS} - V_{TH} \\ \mu C_{OX} \frac{W}{2L}\left(V_{GS} - V_{TH}\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_{TH} \quad V_{DS} \ge V_{GS} - V_{TH} \end{cases}$$

Note: This introduces small discontinuity in model at SAT/Triode transition

Further Model Extensions

Existing model does not depend upon the bulk voltage !

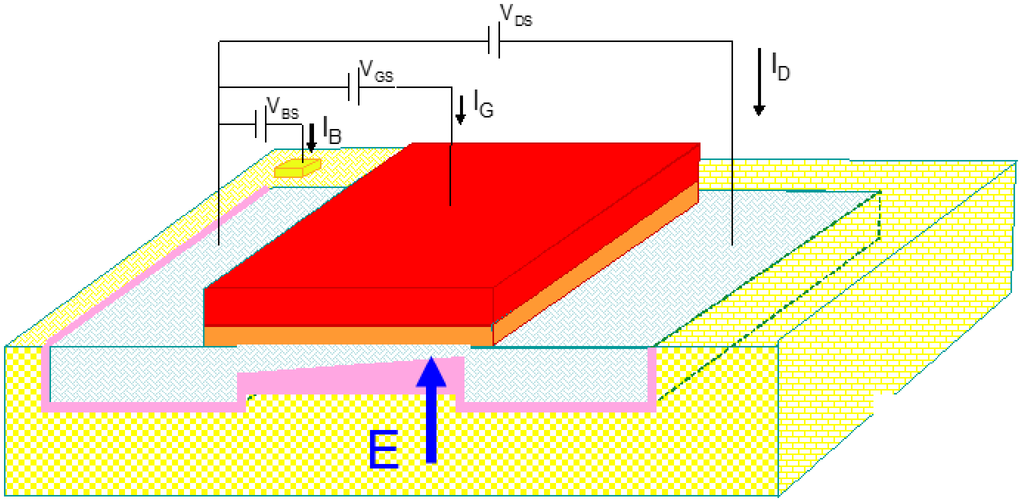
Observe that changing the bulk voltage will change the electric field in the channel region !



Further Model Extensions

Existing model does not depend upon the bulk voltage !

Observe that changing the bulk voltage will change the electric field in the channel region !



Changing the bulk voltage will change the thickness of the inversion layer

Changing the bulk voltage will change the threshold voltage of the device

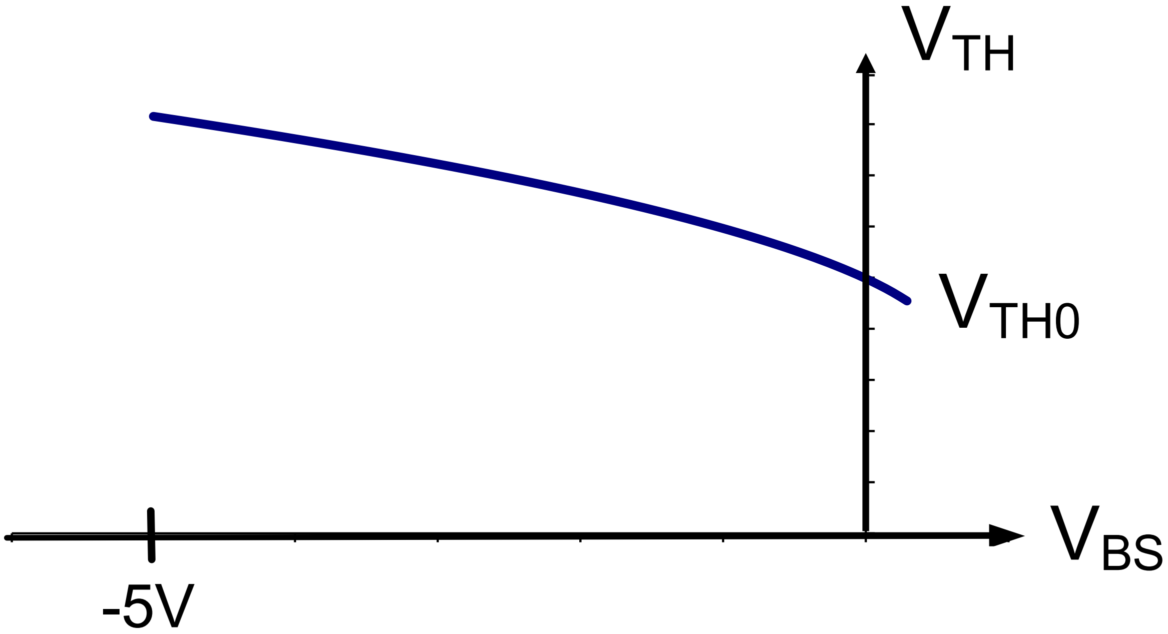
$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

ϕ is the surface potential (some authors use symbol Φ_S)
γ is the bulk threshold

Typical Bulk Effects on Threshold Voltage for n-channel Devices

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

$$\gamma \cong 0.4V^{1/2} \qquad \phi \cong 0.6V$$

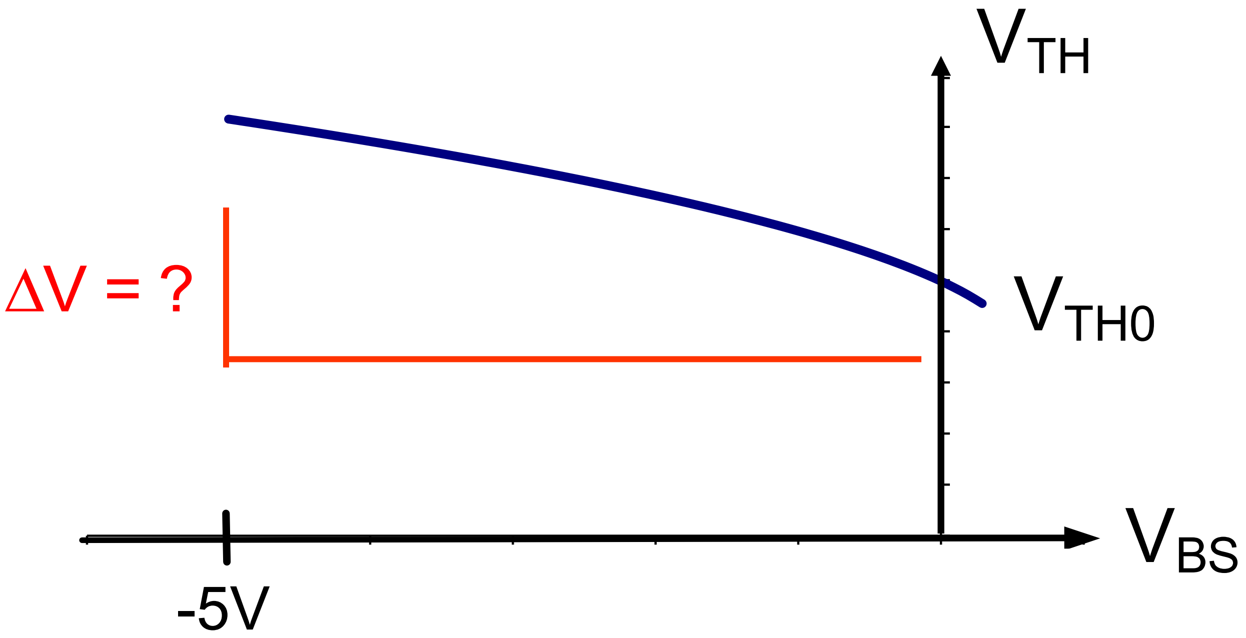


- Bulk-Diffusion Generally Reverse Biased (V_{BS}<0 or at least V_{BS}<0.3V) for n-channel
- Shift in threshold voltage with bulk voltage can be substantial
- Often V_{BS}=0

Typical Bulk Effects on Threshold Voltage for n-channel Devices

$$V_{TH}=V_{TH0}+\gamma\left(\sqrt{\phi-V_{BS}}-\sqrt{\phi}\right)$$

$$\gamma \cong 0.4V^{1/2} \qquad \phi \cong 0.6V$$



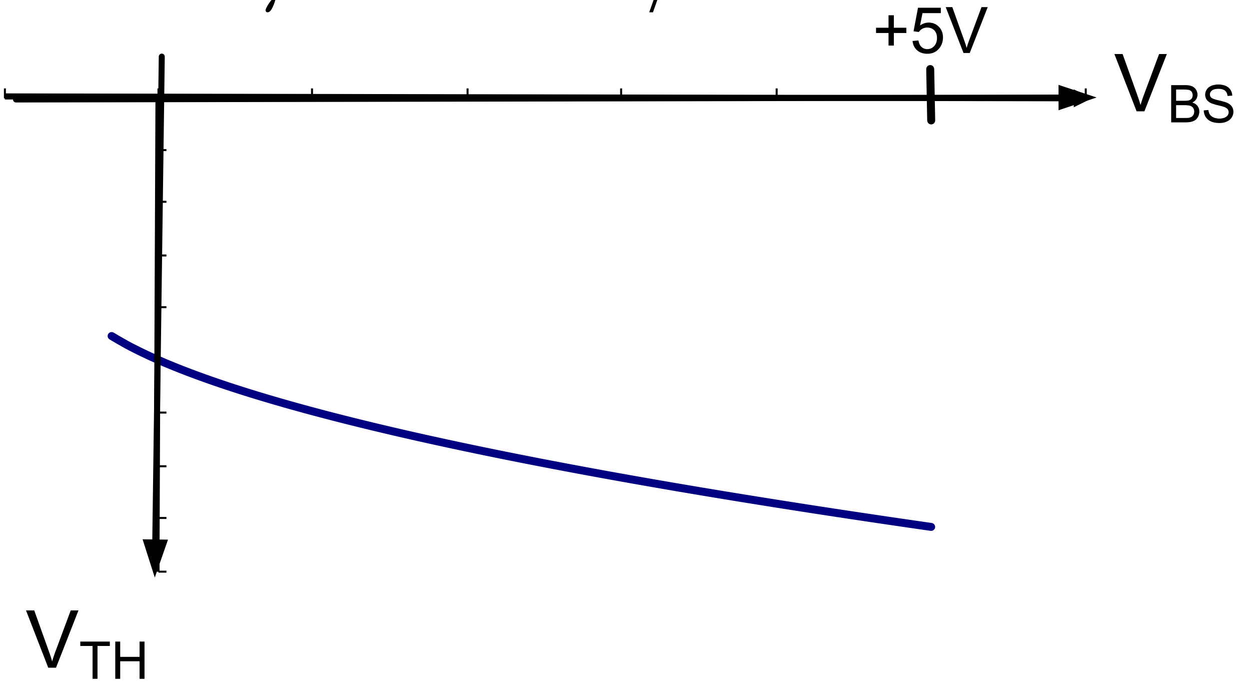
$$\Delta V=V_{TH}-V_{TH0}=\gamma\left(\sqrt{\phi-V_{BS}}-\sqrt{\phi}\right)$$

$$\Delta V \cong 0.4\left(\sqrt{0.6V--5V}-\sqrt{0.6}\right)\cong 0.64V$$

Typical Bulk Effects on Threshold Voltage for p-channel Devices

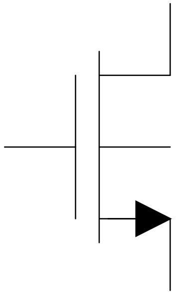
$$V_{TH}=V_{TH0}-\gamma\left(\sqrt{\phi+V_{BS}}-\sqrt{\phi}\right)$$

$$\gamma \cong 0.4V^{1/2} \qquad \phi \cong 0.6V$$



- Bulk-Diffusion Generally Reverse Biased (V_{BS}>0 or at least V_{BS}>-0.3V) for p-channel
- Same functional form as for n-channel but $V_{TH0}<0$
- Magnitude of threshold voltage increases with magnitude of reverse bias

Model Extension Summary



$$\mathbf{I_G = 0}$$

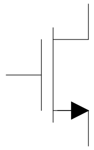
$$\mathbf{I_B = 0}$$

$$I_D = \begin{cases} 0 & V_{GS} \le V_{TH} \\ \mu C_{OX} \frac{W}{L}\left(V_{GS} - V_{TH} - \frac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_{TH} \quad V_{DS} < V_{GS} - V_{TH} \\ \mu C_{OX} \frac{W}{2L}\left(V_{GS} - V_{TH}\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_{TH} \quad V_{DS} \ge V_{GS} - V_{TH} \end{cases}$$

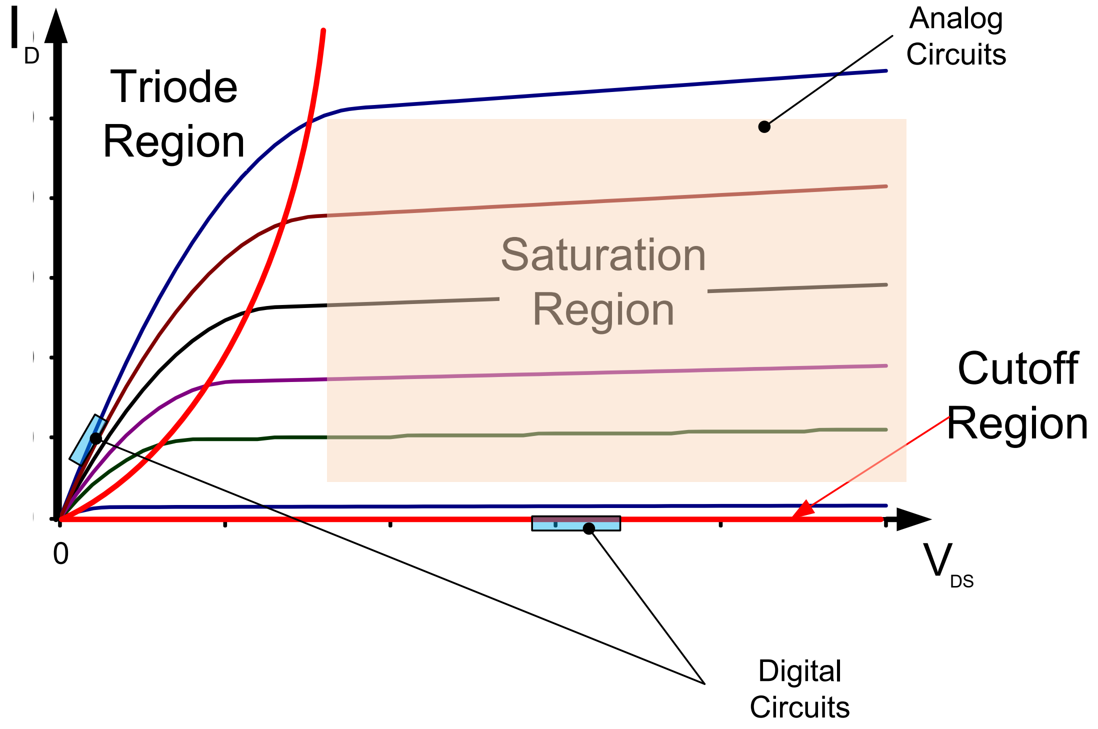
$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{\varphi - V_{BS}} - \sqrt{\varphi}\right)$$

Model Parameters : $\{\mu, C_{OX}, V_{TH0}, \varphi, \gamma, \lambda\}$

Design Parameters : {W,L} but only one degree of freedom W/L



Operation Regions by Applications



Most analog circuits operate in the saturation region

(basic VVR operates in triode and is an exception)

Most digital circuits operate in triode and cutoff regions and switch between these two with Boolean inputs

Model Extension (short devices)

$$I_D = \begin{cases} 0 & V_{GS} \le V_{TH} \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_{TH} - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_{TH} \quad V_{DS} < V_{GS} - V_{TH} \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_{TH}\right)^2 & V_{GS} \ge V_{TH} \quad V_{DS} \ge V_{GS} - V_{TH} \end{cases}$$

As the channel length becomes very short, velocity saturation will occur in the channel and this will occur with electric fields around 2V/u. So, if a gate length is around 1u, then voltages up to 2V can be applied without velocity saturation. But, if gate length decreases and voltages are kept high, velocity saturation will occur

$$I_D = \begin{cases} 0 & V_{GS} \le V_{TH} \\ \dfrac{\theta_2}{\theta_1}\mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_{TH}\right)^{\frac{\alpha}{2}} V_{DS} & V_{GS} \ge V_{TH} \quad V_{DS} < \theta_1\left(V_{GS} - V_{TH}\right)^{\frac{\alpha}{2}} \\ \theta_2 \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_{TH}\right)^{\alpha} & V_{GS} \ge V_{TH} \quad V_{DS} \ge \theta_1\left(V_{GS} - V_{TH}\right)^{\frac{\alpha}{2}} \end{cases}$$

α is the velocity saturation index, $2 \ge \alpha \ge 1$

Model Extension (short devices)

(n-channel device)

$$I_D = \begin{cases} 0 & V_{GS} \le V_{TH} \\ \dfrac{\theta_2}{\theta_1}\mu C_{OX}\dfrac{W}{L}\left(V_{GS}-V_{TH}\right)^{\frac{\alpha}{2}} V_{DS} & V_{GS} \ge V_{TH} \quad V_{DS} < \theta_1\left(V_{GS}-V_{TH}\right)^{\frac{\alpha}{2}} \\ \theta_2\mu C_{OX}\dfrac{W}{L}\left(V_{GS}-V_{TH}\right)^{\alpha} & V_{GS} \ge V_{TH} \quad V_{DS} \ge \theta_1\left(V_{GS}-V_{TH}\right)^{\frac{\alpha}{2}} \end{cases}$$

α is the velocity saturation index, $2 \ge \alpha \ge 1$

No longer a square-law model (some term it an α-power or α-law model)

For long devices, $\alpha=2$

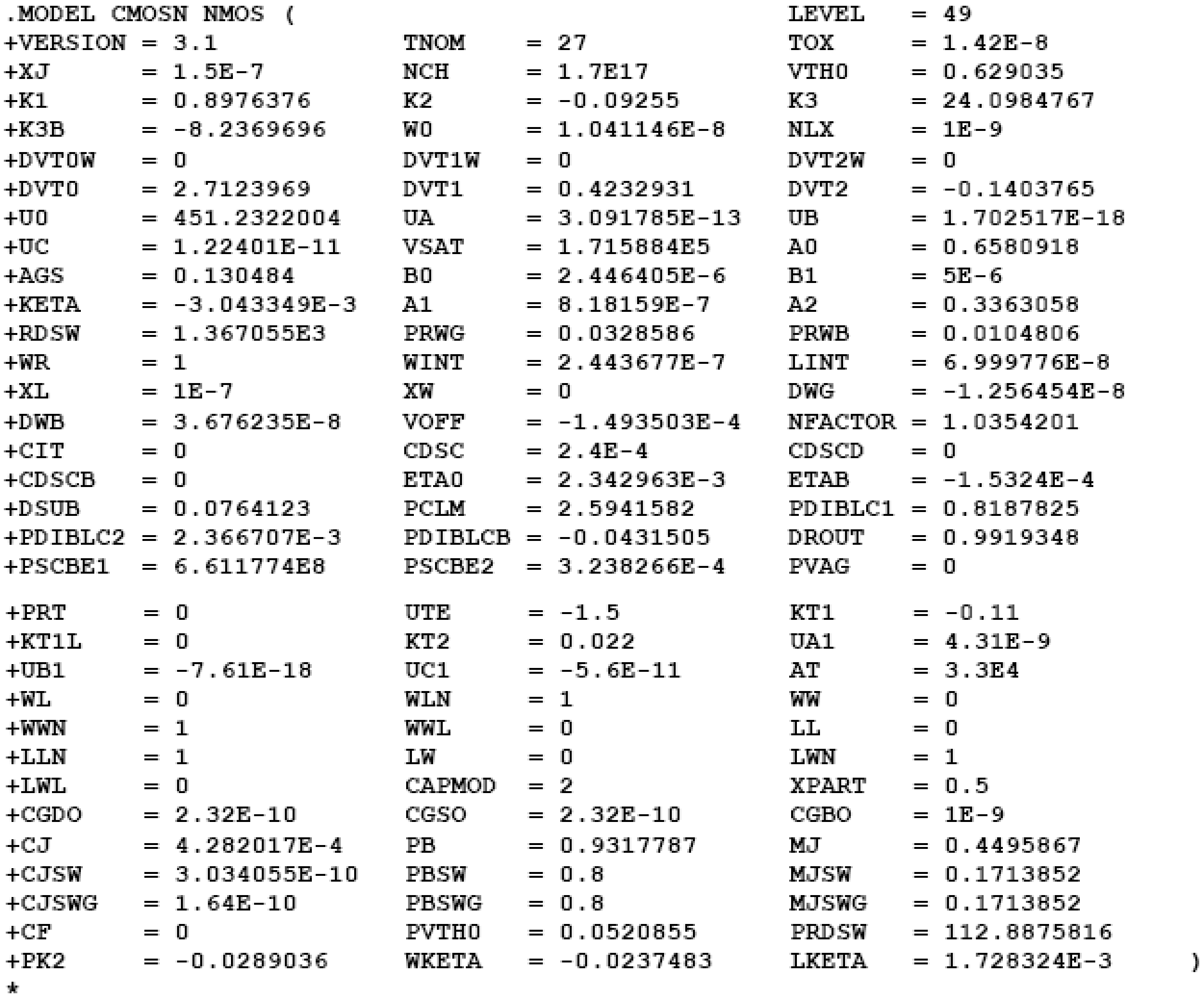
Channel length modulation (λ) and bulk effects can be added to the velocity Saturation as well

Degrading of α is not an attractive limitation of the MOSFET

Be aware of existence but of little use !

(too complicated for analytical calculations, not accurate enough for simulations)

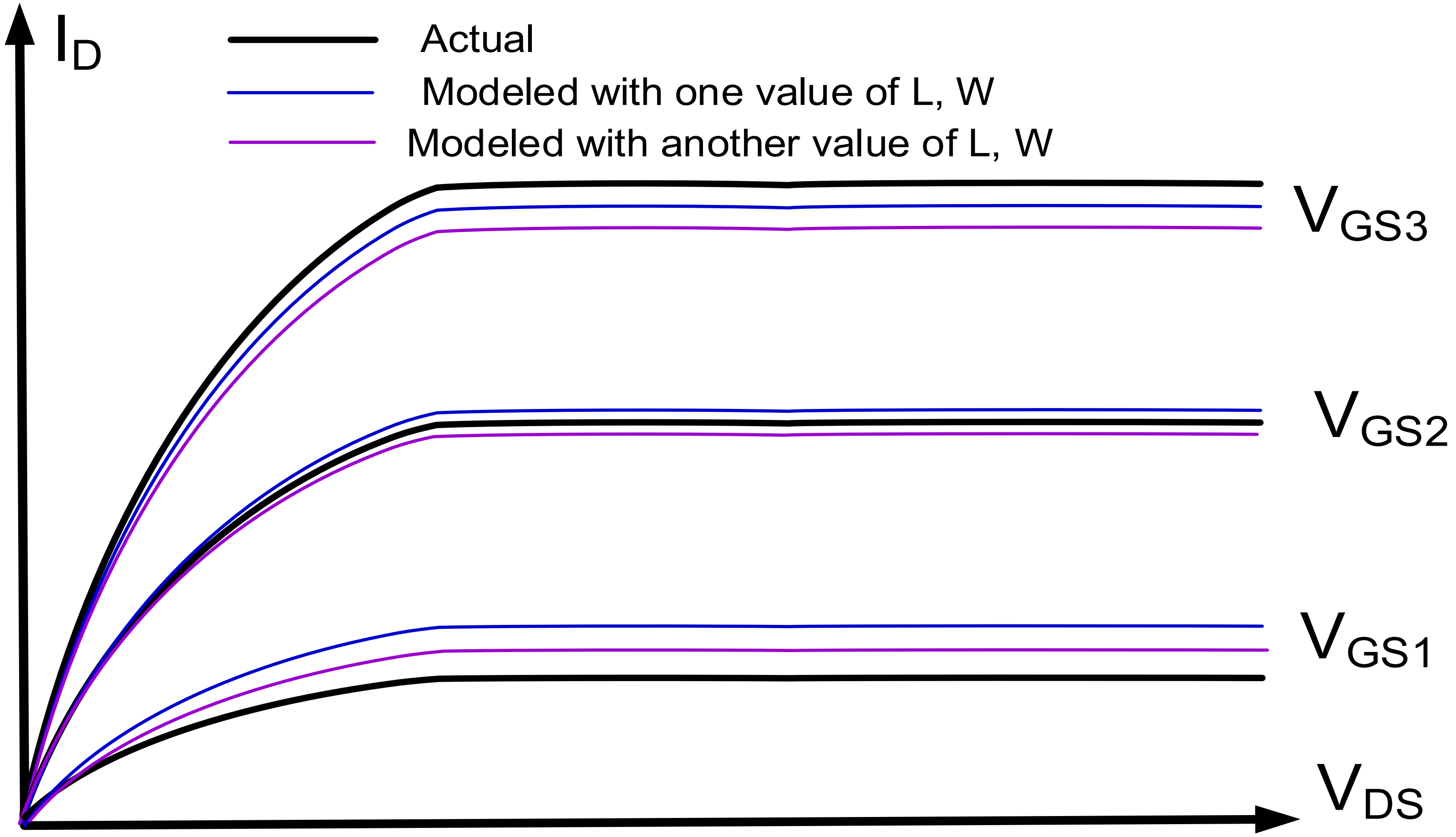
Model Extension (BSIM model)



```
.MODEL CMOSN NMOS (                               LEVEL   = 49
+VERSION = 3.1           TNOM    = 27             TOX     = 1.42E-8
+XJ      = 1.5E-7        NCH     = 1.7E17         VTH0    = 0.629035
+K1      = 0.8976376     K2      = -0.09255       K3      = 24.0984767
+K3B     = -8.2369696    W0      = 1.041146E-8    NLX     = 1E-9
+DVT0W   = 0             DVT1W   = 0              DVT2W   = 0
+DVT0    = 2.7123969     DVT1    = 0.4232931      DVT2    = -0.1403765
+U0      = 451.2322004   UA      = 3.091785E-13   UB      = 1.702517E-18
+UC      = 1.22401E-11   VSAT    = 1.715884E5     A0      = 0.6580918
+AGS     = 0.130484      B0      = 2.446405E-6    B1      = 5E-6
+KETA    = -3.043349E-3  A1      = 8.18159E-7     A2      = 0.3363058
+RDSW    = 1.367055E3    PRWG    = 0.0328586      PRWB    = 0.0104806
+WR      = 1             WINT    = 2.443677E-7    LINT    = 6.999776E-8
+XL      = 1E-7          XW      = 0              DWG     = -1.256454E-8
+DWB     = 3.676235E-8   VOFF    = -1.493503E-4   NFACTOR = 1.0354201
+CIT     = 0             CDSC    = 2.4E-4         CDSCD   = 0
+CDSCB   = 0             ETA0    = 2.342963E-3    ETAB    = -1.5324E-4
+DSUB    = 0.0764123     PCLM    = 2.5941582      PDIBLC1 = 0.8187825
+PDIBLC2 = 2.366707E-3   PDIBLCB = -0.0431505     DROUT   = 0.9919348
+PSCBE1  = 6.611774E8    PSCBE2  = 3.238266E-4    PVAG    = 0

+PRT     = 0             UTE     = -1.5           KT1     = -0.11
+KT1L    = 0             KT2     = 0.022          UA1     = 4.31E-9
+UB1     = -7.61E-18     UC1     = -5.6E-11       AT      = 3.3E4
+WL      = 0             WLN     = 1              WW      = 0
+WWN     = 1             WWL     = 0              LL      = 0
+LLN     = 1             LW      = 0              LWN     = 1
+LWL     = 0             CAPMOD  = 2              XPART   = 0.5
+CGDO    = 2.32E-10      CGSO    = 2.32E-10       CGBO    = 1E-9
+CJ      = 4.282017E-4   PB      = 0.9317787      MJ      = 0.4495867
+CJSW    = 3.034055E-10  PBSW    = 0.8            MJSW    = 0.1713852
+CJSWG   = 1.64E-10      PBSWG   = 0.8            MJSWG   = 0.1713852
+CF      = 0             PVTH0   = 0.0520855      PRDSW   = 112.8875816
+PK2     = -0.0289036    WKETA   = -0.0237483     LKETA   = 1.728324E-3      )
*
```

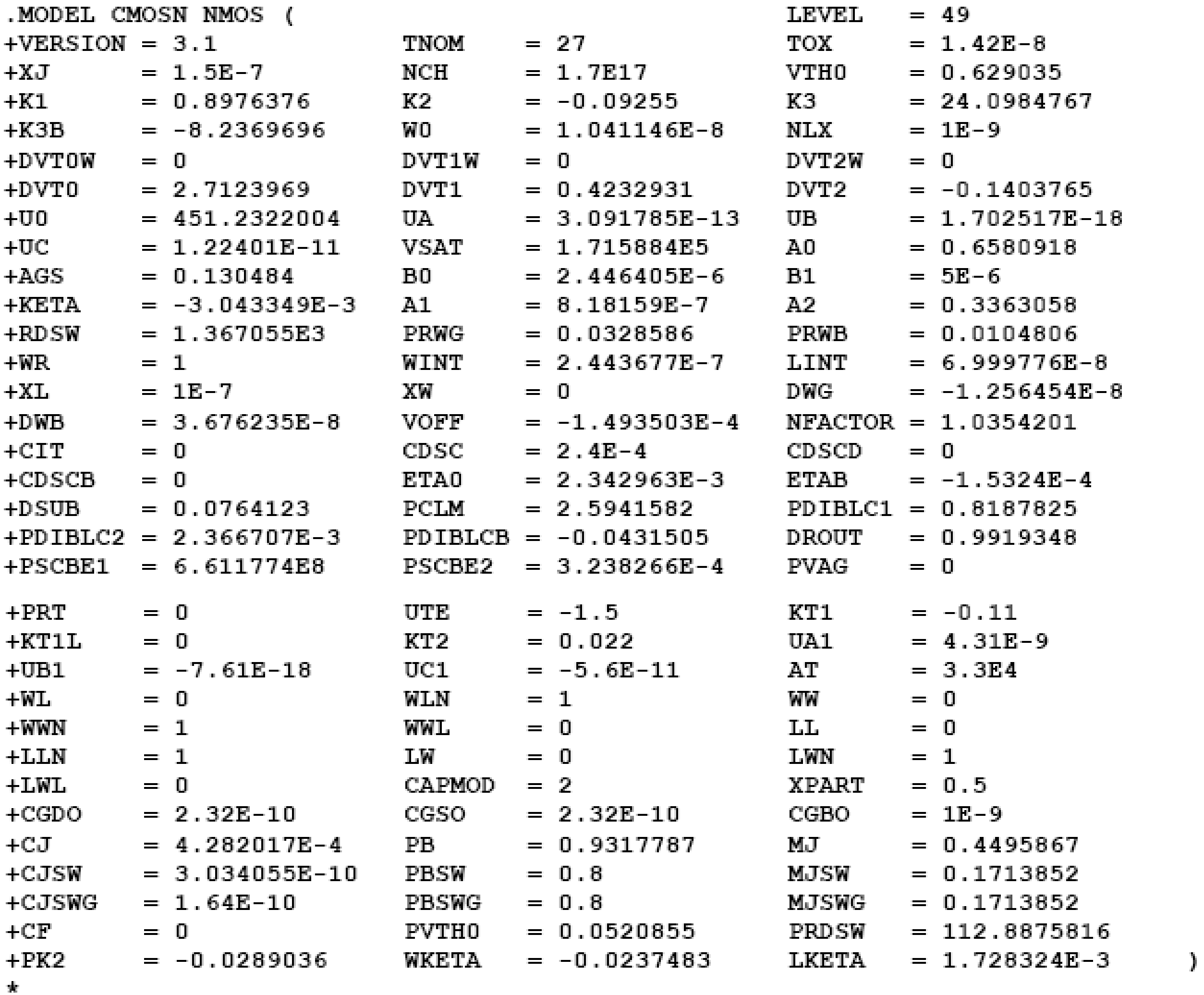
Model Errors with Different W/L Values



Binning models can improve model accuracy

BSIM Binning Model

- Bin on device sizes
- multiple BSIM models !



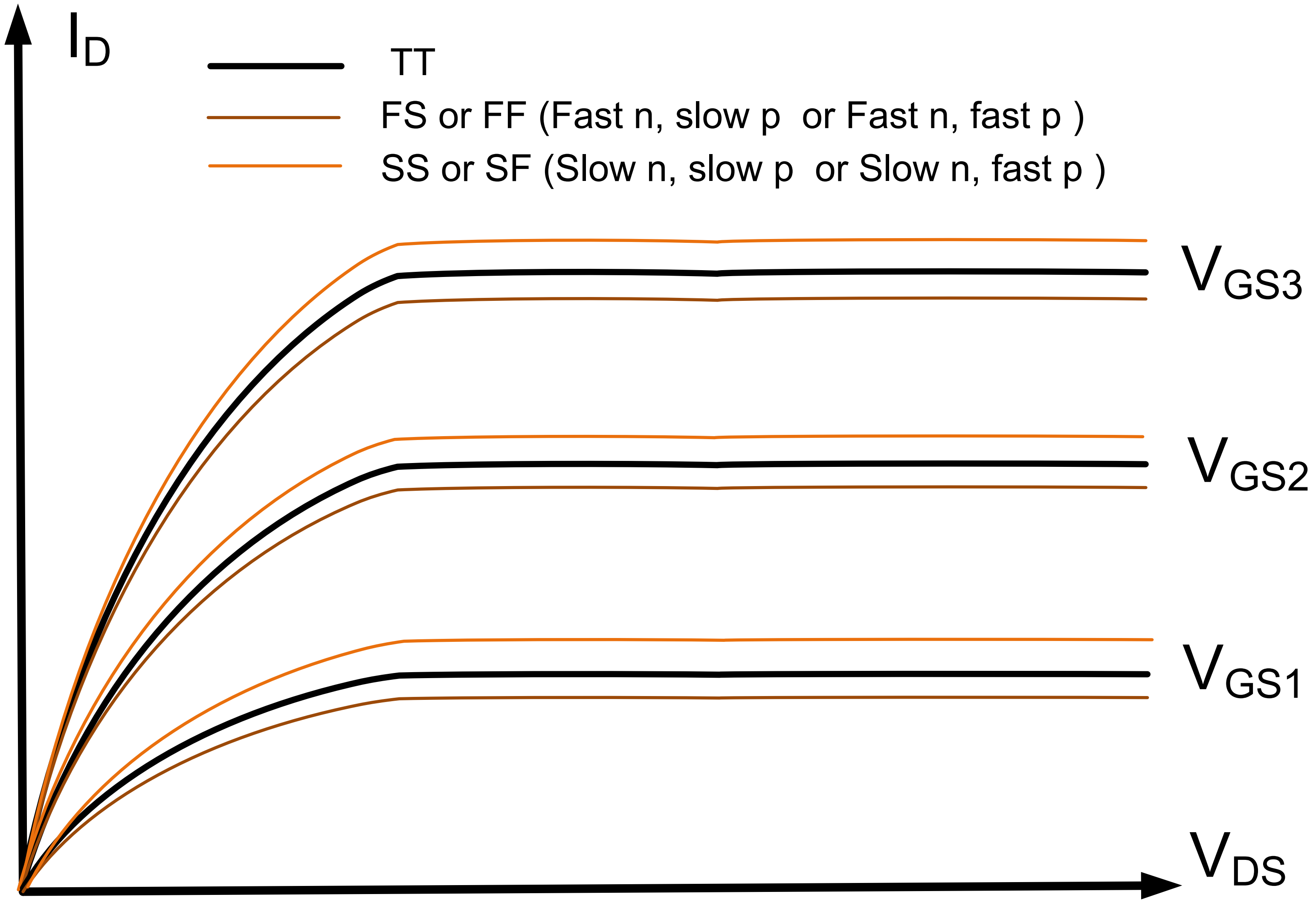
```
.MODEL CMOSN NMOS (                                    LEVEL   = 49
+VERSION = 3.1            TNOM    = 27                 TOX     = 1.42E-8
+XJ      = 1.5E-7         NCH     = 1.7E17             VTH0    = 0.629035
+K1      = 0.8976376      K2      = -0.09255           K3      = 24.0984767
+K3B     = -8.2369696     W0      = 1.041146E-8        NLX     = 1E-9
+DVT0W   = 0              DVT1W   = 0                  DVT2W   = 0
+DVT0    = 2.7123969      DVT1    = 0.4232931          DVT2    = -0.1403765
+U0      = 451.2322004    UA      = 3.091785E-13       UB      = 1.702517E-18
+UC      = 1.22401E-11    VSAT    = 1.715884E5         A0      = 0.6580918
+AGS     = 0.130484       B0      = 2.446405E-6        B1      = 5E-6
+KETA    = -3.043349E-3   A1      = 8.18159E-7         A2      = 0.3363058
+RDSW    = 1.367055E3     PRWG    = 0.0328586          PRWB    = 0.0104806
+WR      = 1              WINT    = 2.443677E-7        LINT    = 6.999776E-8
+XL      = 1E-7           XW      = 0                  DWG     = -1.256454E-8
+DWB     = 3.676235E-8    VOFF    = -1.493503E-4       NFACTOR = 1.0354201
+CIT     = 0              CDSC    = 2.4E-4             CDSCD   = 0
+CDSCB   = 0              ETA0    = 2.342963E-3        ETAB    = -1.5324E-4
+DSUB    = 0.0764123      PCLM    = 2.5941582          PDIBLC1 = 0.8187825
+PDIBLC2 = 2.366707E-3    PDIBLCB = -0.0431505         DROUT   = 0.9919348
+PSCBE1  = 6.611774E8     PSCBE2  = 3.238266E-4        PVAG    = 0

+PRT     = 0              UTE     = -1.5               KT1     = -0.11
+KT1L    = 0              KT2     = 0.022              UA1     = 4.31E-9
+UB1     = -7.61E-18      UC1     = -5.6E-11           AT      = 3.3E4
+WL      = 0              WLN     = 1                  WW      = 0
+WWN     = 1              WWL     = 0                  LL      = 0
+LLN     = 1              LW      = 0                  LWN     = 1
+LWL     = 0              CAPMOD  = 2                  XPART   = 0.5
+CGDO    = 2.32E-10       CGSO    = 2.32E-10           CGBO    = 1E-9
+CJ      = 4.282017E-4    PB      = 0.9317787          MJ      = 0.4495867
+CJSW    = 3.034055E-10   PBSW    = 0.8                MJSW    = 0.1713852
+CJSWG   = 1.64E-10       PBSWG   = 0.8                MJSWG   = 0.1713852
+CF      = 0              PVTH0   = 0.0520855          PRDSW   = 112.8875816
+PK2     = -0.0289036     WKETA   = -0.0237483         LKETA   = 1.728324E-3      )
*
```

With 32 bins, this model has 3040 model parameters !

Model Changes with Process Variations

(n-ch characteristics shown)

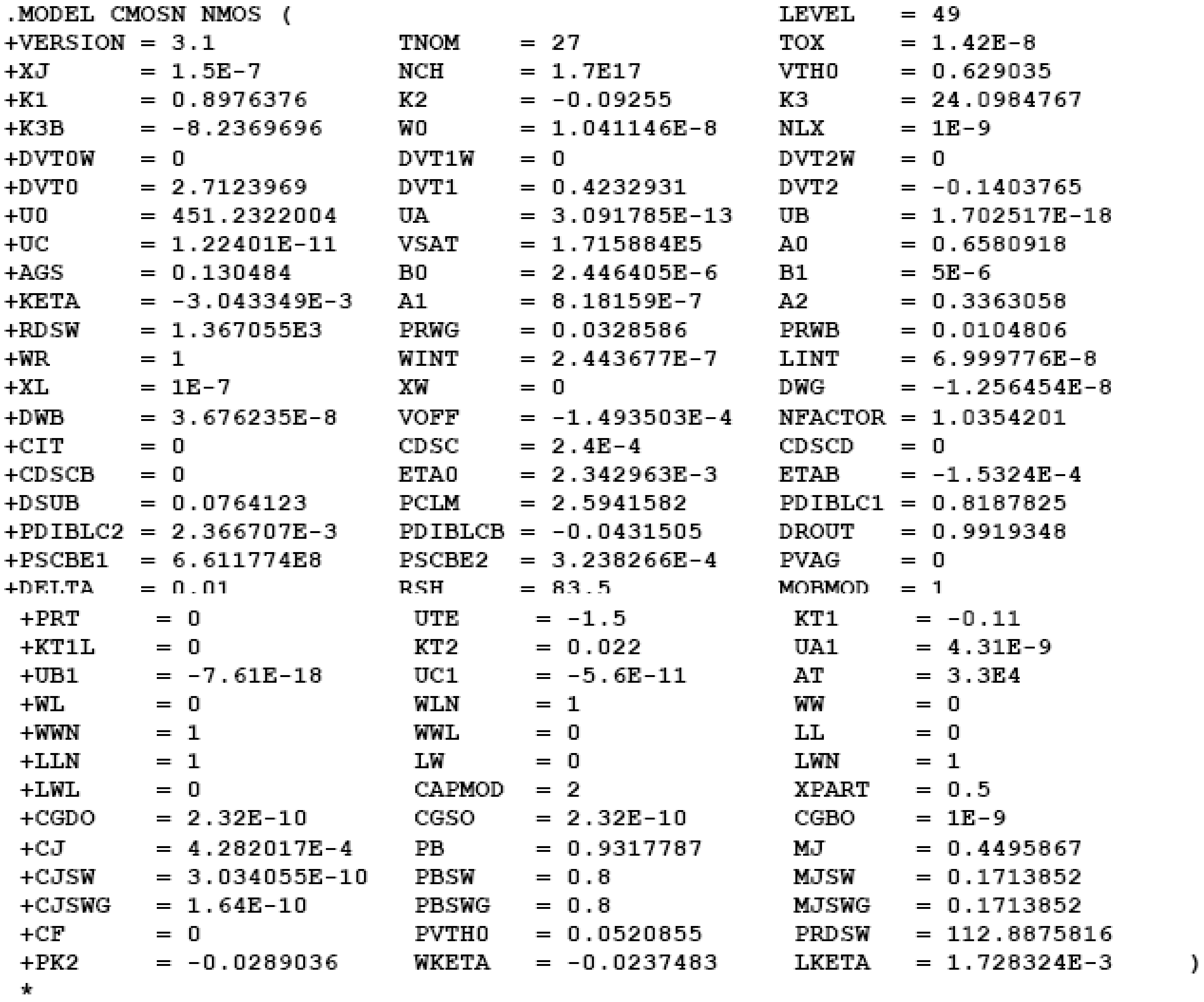


Corner models can improve model accuracy

BSIM Corner Models with Binning

- Often 4 corners in addition to nominal TT, FF, FS, SF, and SS

- bin on device sizes



```
.MODEL CMOSN NMOS (                                LEVEL    = 49
+VERSION = 3.1            TNOM     = 27            TOX      = 1.42E-8
+XJ      = 1.5E-7         NCH      = 1.7E17        VTH0     = 0.629035
+K1      = 0.8976376      K2       = -0.09255      K3       = 24.0984767
+K3B     = -8.2369696     W0       = 1.041146E-8   NLX      = 1E-9
+DVT0W   = 0              DVT1W    = 0             DVT2W    = 0
+DVT0    = 2.7123969      DVT1     = 0.4232931     DVT2     = -0.1403765
+U0      = 451.2322004    UA       = 3.091785E-13  UB       = 1.702517E-18
+UC      = 1.22401E-11    VSAT     = 1.715884E5    A0       = 0.6580918
+AGS     = 0.130484       B0       = 2.446405E-6   B1       = 5E-6
+KETA    = -3.043349E-3   A1       = 8.18159E-7    A2       = 0.3363058
+RDSW    = 1.367055E3     PRWG     = 0.0328586     PRWB     = 0.0104806
+WR      = 1              WINT     = 2.443677E-7   LINT     = 6.999776E-8
+XL      = 1E-7           XW       = 0             DWG      = -1.256454E-8
+DWB     = 3.676235E-8    VOFF     = -1.493503E-4  NFACTOR  = 1.0354201
+CIT     = 0              CDSC     = 2.4E-4        CDSCD    = 0
+CDSCB   = 0              ETA0     = 2.342963E-3   ETAB     = -1.5324E-4
+DSUB    = 0.0764123      PCLM     = 2.5941582     PDIBLC1  = 0.8187825
+PDIBLC2 = 2.366707E-3    PDIBLCB  = -0.0431505    DROUT    = 0.9919348
+PSCBE1  = 6.611774E8     PSCBE2   = 3.238266E-4   PVAG     = 0
+DELTA   = 0.01           RSH      = 83.5          MOBMOD   = 1
 +PRT      = 0             UTE      = -1.5          KT1      = -0.11
 +KT1L     = 0             KT2      = 0.022         UA1      = 4.31E-9
 +UB1      = -7.61E-18     UC1      = -5.6E-11      AT       = 3.3E4
 +WL       = 0             WLN      = 1             WW       = 0
 +WWN      = 1             WWL      = 0             LL       = 0
 +LLN      = 1             LW       = 0             LWN      = 1
 +LWL      = 0             CAPMOD   = 2             XPART    = 0.5
 +CGDO     = 2.32E-10      CGSO     = 2.32E-10      CGBO     = 1E-9
 +CJ       = 4.282017E-4   PB       = 0.9317787     MJ       = 0.4495867
 +CJSW     = 3.034055E-10  PBSW     = 0.8           MJSW     = 0.1713852
 +CJSWG    = 1.64E-10      PBSWG    = 0.8           MJSWG    = 0.1713852
 +CF       = 0             PVTH0    = 0.0520855     PRDSW    = 112.8875816
 +PK2      = -0.0289036    WKETA    = -0.0237483    LKETA    = 1.728324E-3     )
 *
```

With 32 size bins and 4 corners, this model has 15,200 model parameters !

How many models of the MOSFET do we have?

Switch-level model (2)

Square-law model

Square-law model (with λ and bulk additions)

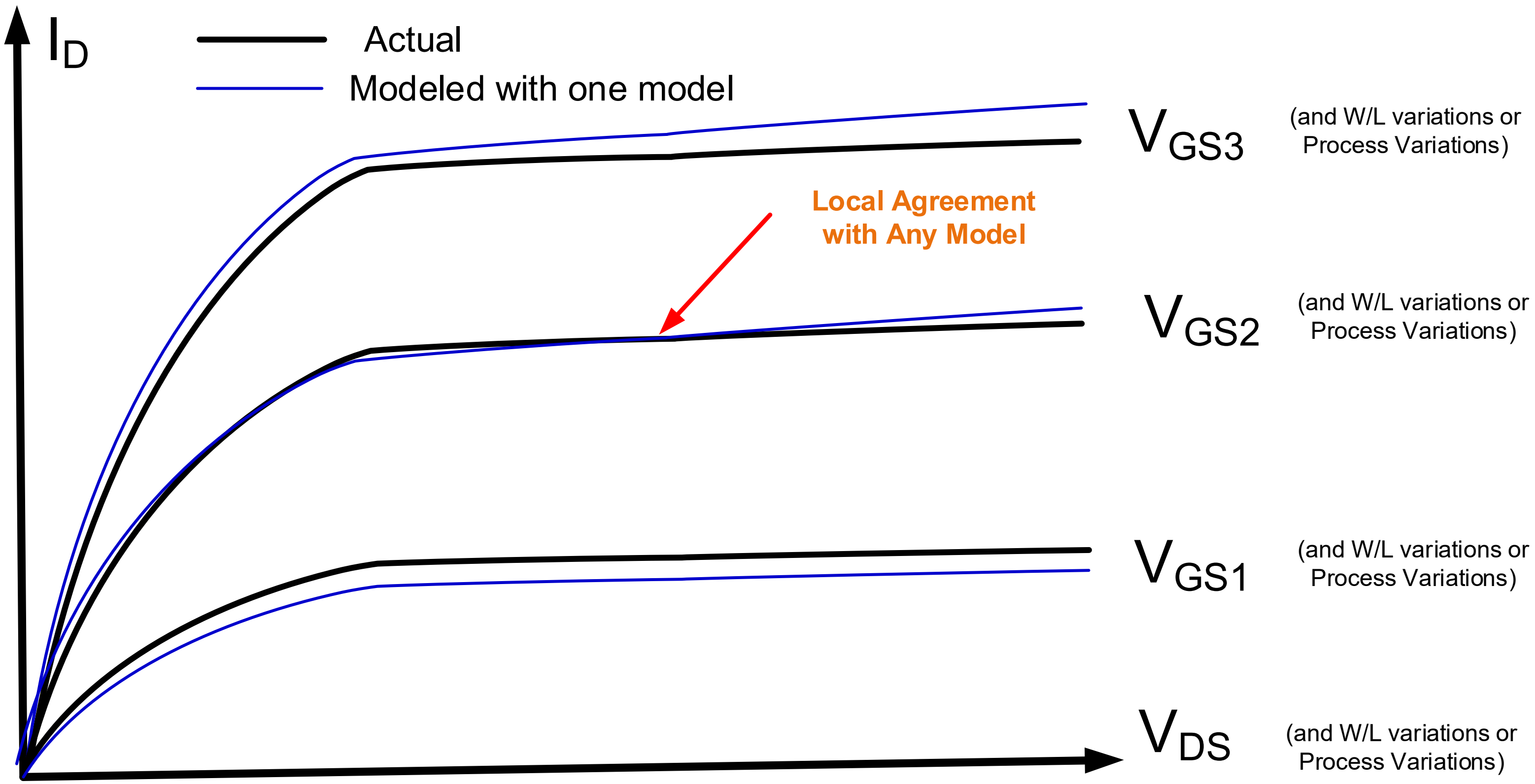
α-law model (with λ and bulk additions)

BSIM model

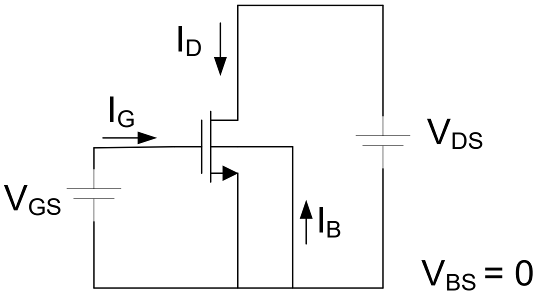
BSIM model (with binning extensions)

BSIM model (with binning extensions and process corners)

The Modeling Challenge



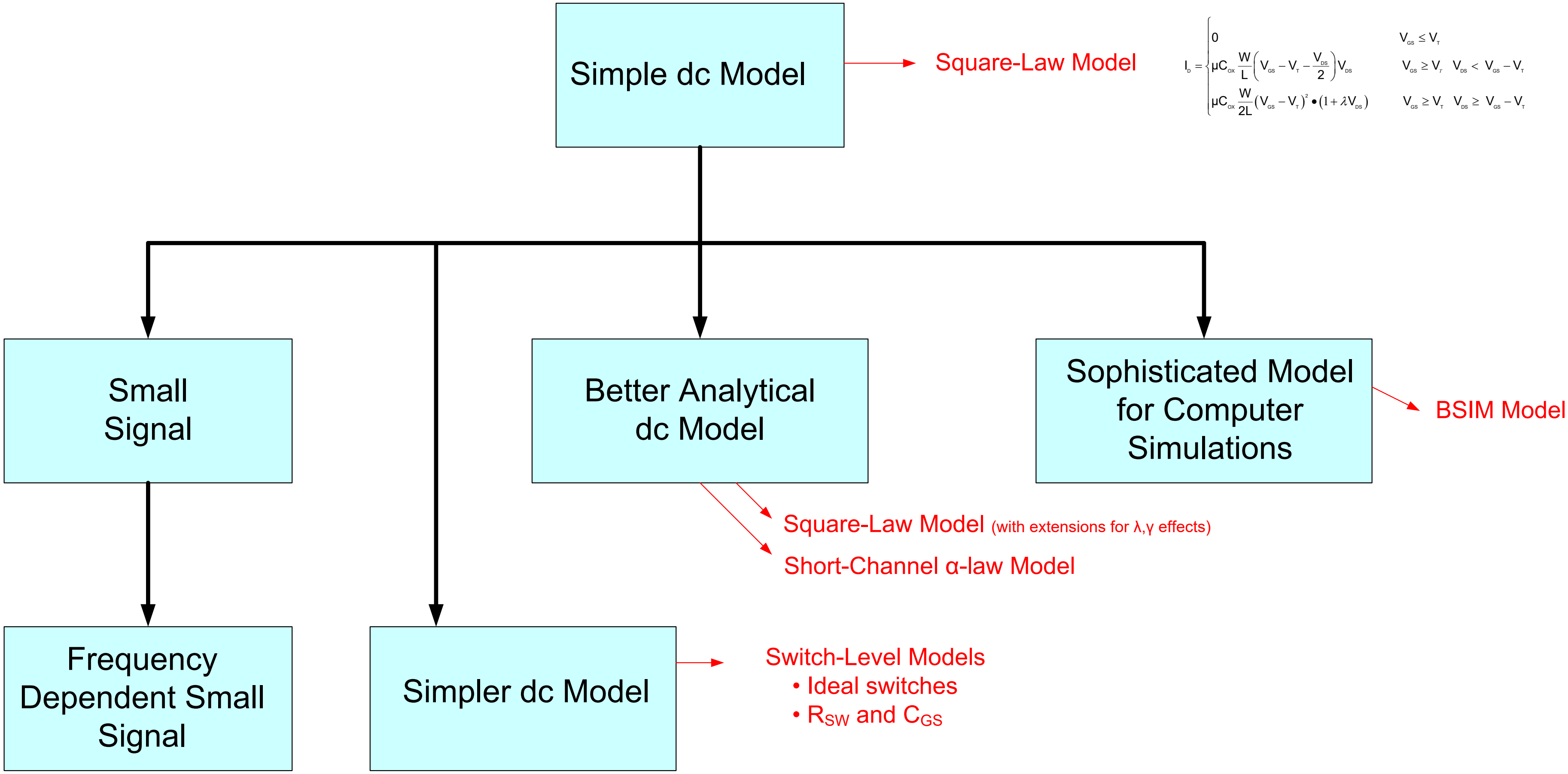
Local Agreement with Any Model



$$I_D = f_1(V_{GS}, V_{DS})$$
$$I_G = f_2(V_{GS}, V_{DS})$$
$$I_B = f_3(V_{GS}, V_{DS})$$

Difficult to obtain analytical functions that accurately fit actual devices over bias, size, and process variations

Model Status



Simple dc Model

Square-Law Model

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \frac{W}{L}\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^2 \bullet (1 + \lambda V_{DS}) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

Small Signal

Better Analytical dc Model

Sophisticated Model for Computer Simulations

BSIM Model

Square-Law Model (with extensions for λ,γ effects)

Short-Channel α-law Model

Frequency Dependent Small Signal

Simpler dc Model

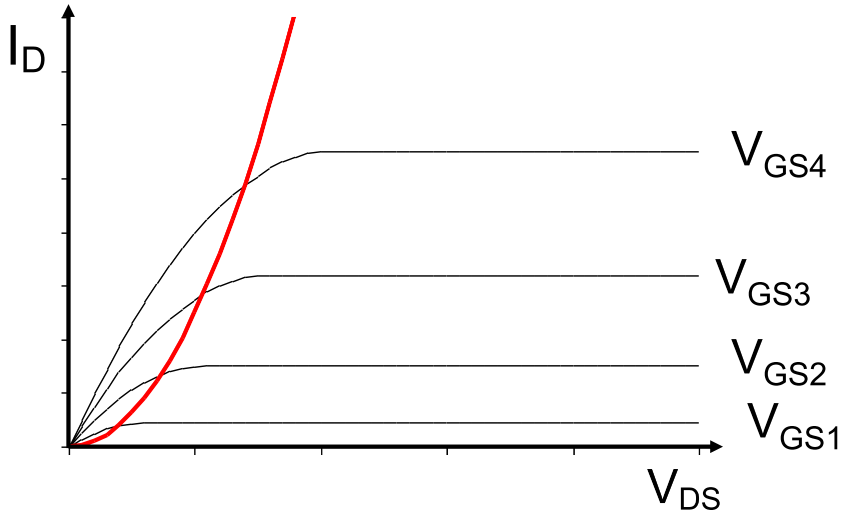
Switch-Level Models

- Ideal switches
- R_{SW} and C_{GS}

In the next few slides, the models we have developed will be listed and reviewed

- Square-law Model
- Switch-level Models
- Extended Square-law model
- Short-channel model
- BSIM Model
- BSIM Binning Model
- Corner Models

Square-Law Model

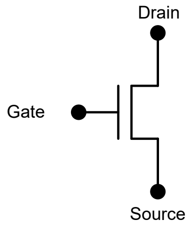


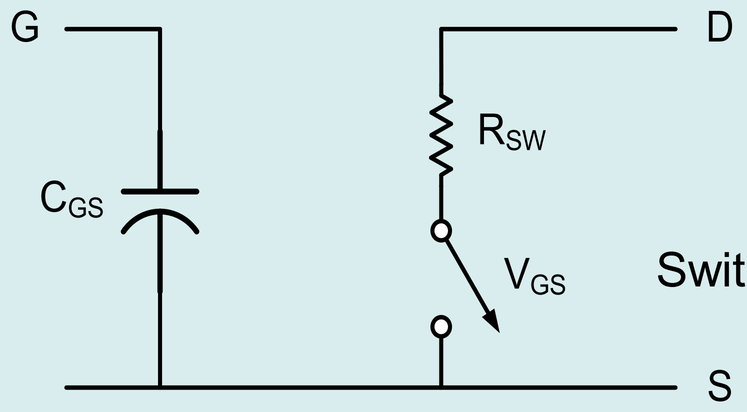
$$I_D = \begin{cases} 0 & V_{GS} \le V_{TH} \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_{TH} - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_{TH} \quad V_{DS} < V_{GS} - V_{TH} \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_{TH}\right)^2 & V_{GS} \ge V_{TH} \quad V_{DS} \ge V_{GS} - V_{TH} \end{cases}$$

Model Parameters : $\{\mu, C_{OX}, V_{TH0}\}$

Design Parameters : {W,L}   but only one degree of freedom W/L

Switch-Level Models





Switch closed for V_{GS}="1"

Switch-level model including gate capacitance and drain resistance

C_{GS} and R_{SW} dependent upon device sizes and process

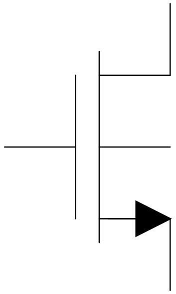
For minimum-sized devices in a 0.5u process

$$\mathbf{C_{GS} \cong 1.5fF} \qquad \mathbf{R_{sw}} \cong \left.\begin{matrix} \mathbf{2K\Omega\ \ n-channel} \\ \mathbf{6K\Omega\ \ p-channel} \end{matrix}\right\}$$

Considerable emphasis will be placed upon device sizing to manage C_{GS} and R_{SW}

Model Parameters : $\{C_{GS},R_{SW}\}$

Extended Square-Law Model



$$\mathbf{I_G = 0}$$

$$\mathbf{I_B = 0}$$

$$I_D = \begin{cases} 0 & V_{GS} \le V_{TH} \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_{TH} - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_{TH} \quad V_{DS} < V_{GS} - V_{TH} \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_{TH}\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_{TH} \quad V_{DS} \ge V_{GS} - V_{TH} \end{cases}$$

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{\varphi - V_{BS}} - \sqrt{\varphi}\right)$$

Model Parameters : $\{\mu, C_{OX}, V_{TH0}, \varphi, \gamma, \lambda\}$

Design Parameters : $\{W,L\}$ but only one degree of freedom W/L

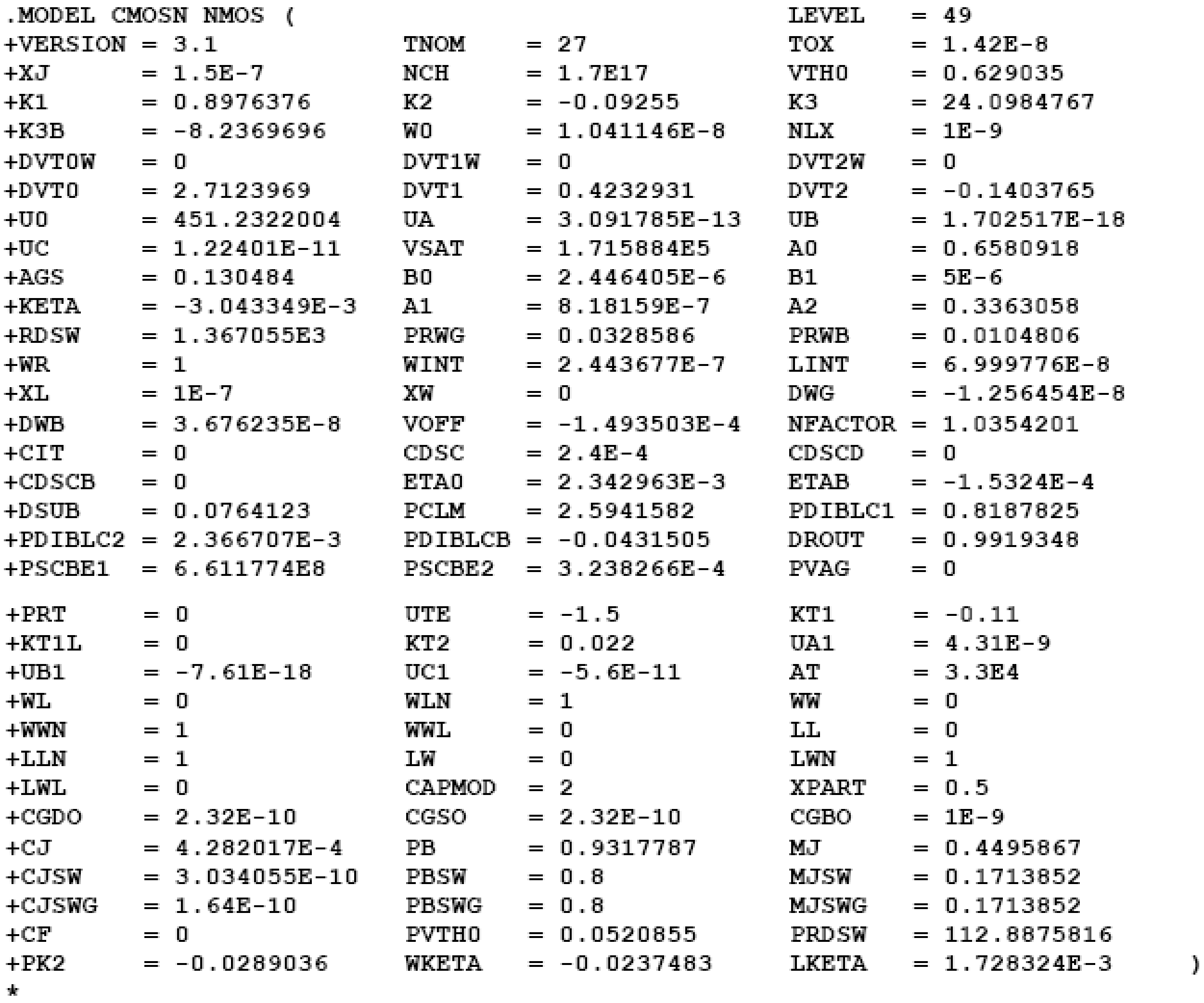
Short-Channel Model

$$I_D = \begin{cases} 0 & V_{GS} \le V_{TH} \\ \dfrac{\theta_2}{\theta_1}\mu C_{OX}\dfrac{W}{L}\left(V_{GS}-V_{TH}\right)^{\frac{\alpha}{2}} V_{DS} & V_{GS} \ge V_{TH} \quad V_{DS} < \theta_1\left(V_{GS}-V_{TH}\right)^{\frac{\alpha}{2}} \\ \theta_2\mu C_{OX}\dfrac{W}{L}\left(V_{GS}-V_{TH}\right)^{\alpha} & V_{GS} \ge V_{TH} \quad V_{DS} \ge \theta_1\left(V_{GS}-V_{TH}\right)^{\frac{\alpha}{2}} \end{cases}$$

α is the velocity saturation index, $2 \ge \alpha \ge 1$

Channel length modulation (λ) and bulk effects can be added to the velocity Saturation as well

BSIM model



```
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+DVT0W   = 0             DVT1W   = 0                   DVT2W   = 0
+DVT0    = 2.7123969     DVT1    = 0.4232931           DVT2    = -0.1403765
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+UC      = 1.22401E-11   VSAT    = 1.715884E5          A0      = 0.6580918
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+CIT     = 0             CDSC    = 2.4E-4              CDSCD   = 0
+CDSCB   = 0             ETA0    = 2.342963E-3         ETAB    = -1.5324E-4
+DSUB    = 0.0764123     PCLM    = 2.5941582           PDIBLC1 = 0.8187825
+PDIBLC2 = 2.366707E-3   PDIBLCB = -0.0431505          DROUT   = 0.9919348
+PSCBE1  = 6.611774E8    PSCBE2  = 3.238266E-4         PVAG    = 0

+PRT     = 0             UTE     = -1.5                KT1     = -0.11
+KT1L    = 0             KT2     = 0.022               UA1     = 4.31E-9
+UB1     = -7.61E-18     UC1     = -5.6E-11            AT      = 3.3E4
+WL      = 0             WLN     = 1                   WW      = 0
+WWN     = 1             WWL     = 0                   LL      = 0
+LLN     = 1             LW      = 0                   LWN     = 1
+LWL     = 0             CAPMOD  = 2                   XPART   = 0.5
+CGDO    = 2.32E-10      CGSO    = 2.32E-10            CGBO    = 1E-9
+CJ      = 4.282017E-4   PB      = 0.9317787           MJ      = 0.4495867
+CJSW    = 3.034055E-10  PBSW    = 0.8                 MJSW    = 0.1713852
+CJSWG   = 1.64E-10      PBSWG   = 0.8                 MJSWG   = 0.1713852
+CF      = 0             PVTH0   = 0.0520855           PRDSW   = 112.8875816
+PK2     = -0.0289036    WKETA   = -0.0237483          LKETA   = 1.728324E-3     )
*
```
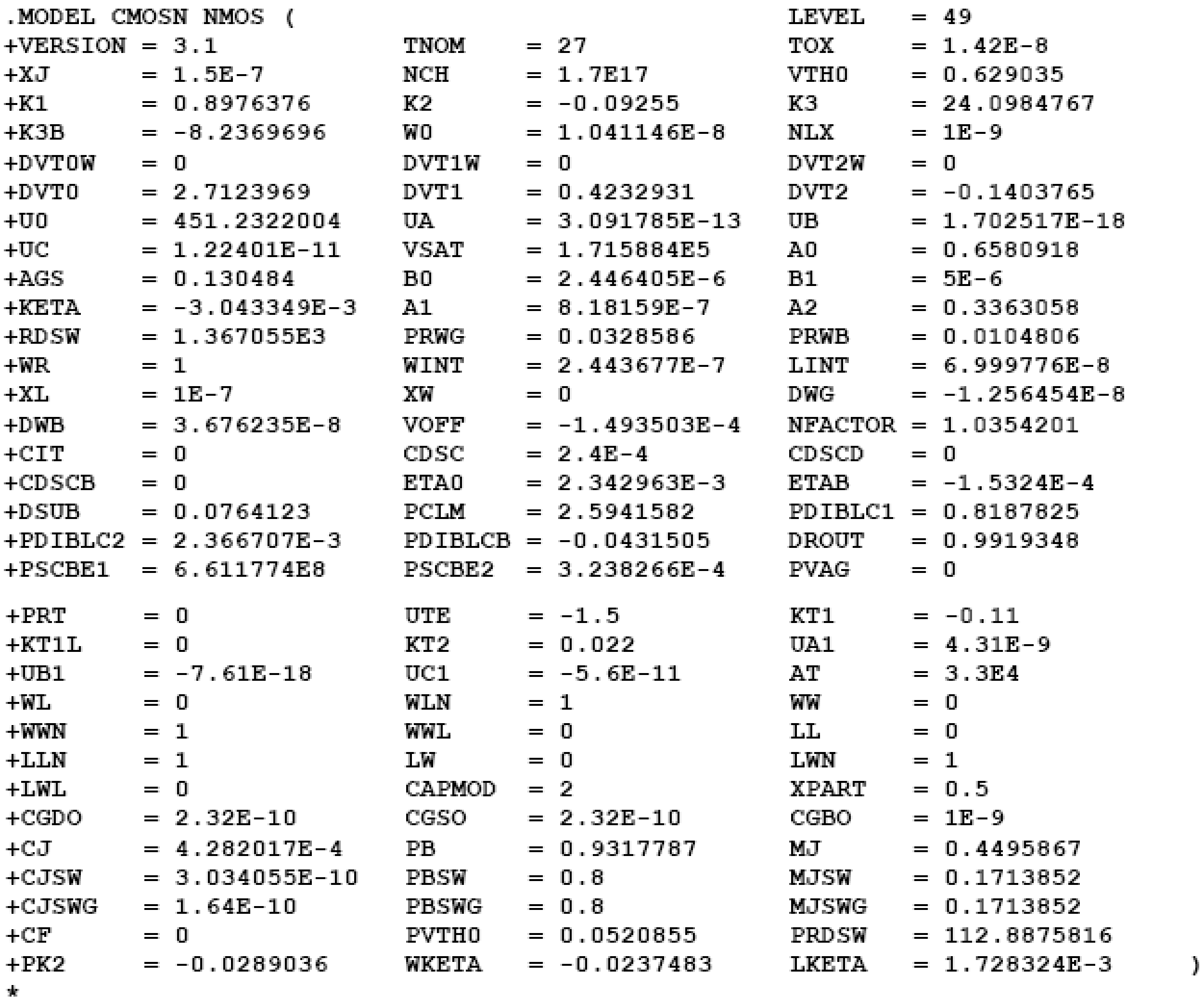

Note this model has 95 model parameters !

BSIM Binning Model

- Bin on device sizes
- multiple BSIM models !



```
.MODEL CMOSN NMOS (                                      LEVEL    = 49
+VERSION = 3.1            TNOM     = 27                  TOX      = 1.42E-8
+XJ      = 1.5E-7         NCH      = 1.7E17              VTH0     = 0.629035
+K1      = 0.8976376      K2       = -0.09255            K3       = 24.0984767
+K3B     = -8.2369696     W0       = 1.041146E-8         NLX      = 1E-9
+DVT0W   = 0              DVT1W    = 0                   DVT2W    = 0
+DVT0    = 2.7123969      DVT1     = 0.4232931           DVT2     = -0.1403765
+U0      = 451.2322004    UA       = 3.091785E-13        UB       = 1.702517E-18
+UC      = 1.22401E-11    VSAT     = 1.715884E5          A0       = 0.6580918
+AGS     = 0.130484       B0       = 2.446405E-6         B1       = 5E-6
+KETA    = -3.043349E-3   A1       = 8.18159E-7          A2       = 0.3363058
+RDSW    = 1.367055E3     PRWG     = 0.0328586           PRWB     = 0.0104806
+WR      = 1              WINT     = 2.443677E-7         LINT     = 6.999776E-8
+XL      = 1E-7           XW       = 0                   DWG      = -1.256454E-8
+DWB     = 3.676235E-8    VOFF     = -1.493503E-4        NFACTOR = 1.0354201
+CIT     = 0              CDSC     = 2.4E-4              CDSCD    = 0
+CDSCB   = 0              ETA0     = 2.342963E-3         ETAB     = -1.5324E-4
+DSUB    = 0.0764123      PCLM     = 2.5941582           PDIBLC1 = 0.8187825
+PDIBLC2 = 2.366707E-3    PDIBLCB = -0.0431505           DROUT    = 0.9919348
+PSCBE1  = 6.611774E8     PSCBE2   = 3.238266E-4         PVAG     = 0

+PRT     = 0              UTE      = -1.5                KT1      = -0.11
+KT1L    = 0              KT2      = 0.022               UA1      = 4.31E-9
+UB1     = -7.61E-18      UC1      = -5.6E-11            AT       = 3.3E4
+WL      = 0              WLN      = 1                   WW       = 0
+WWN     = 1              WWL      = 0                   LL       = 0
+LLN     = 1              LW       = 0                   LWN      = 1
+LWL     = 0              CAPMOD   = 2                   XPART    = 0.5
+CGDO    = 2.32E-10       CGSO     = 2.32E-10            CGBO     = 1E-9
+CJ      = 4.282017E-4    PB       = 0.9317787           MJ       = 0.4495867
+CJSW    = 3.034055E-10   PBSW     = 0.8                 MJSW     = 0.1713852
+CJSWG   = 1.64E-10       PBSWG    = 0.8                 MJSWG    = 0.1713852
+CF      = 0              PVTH0    = 0.0520855           PRDSW    = 112.8875816
+PK2     = -0.0289036     WKETA    = -0.0237483          LKETA    = 1.728324E-3     )
*
```

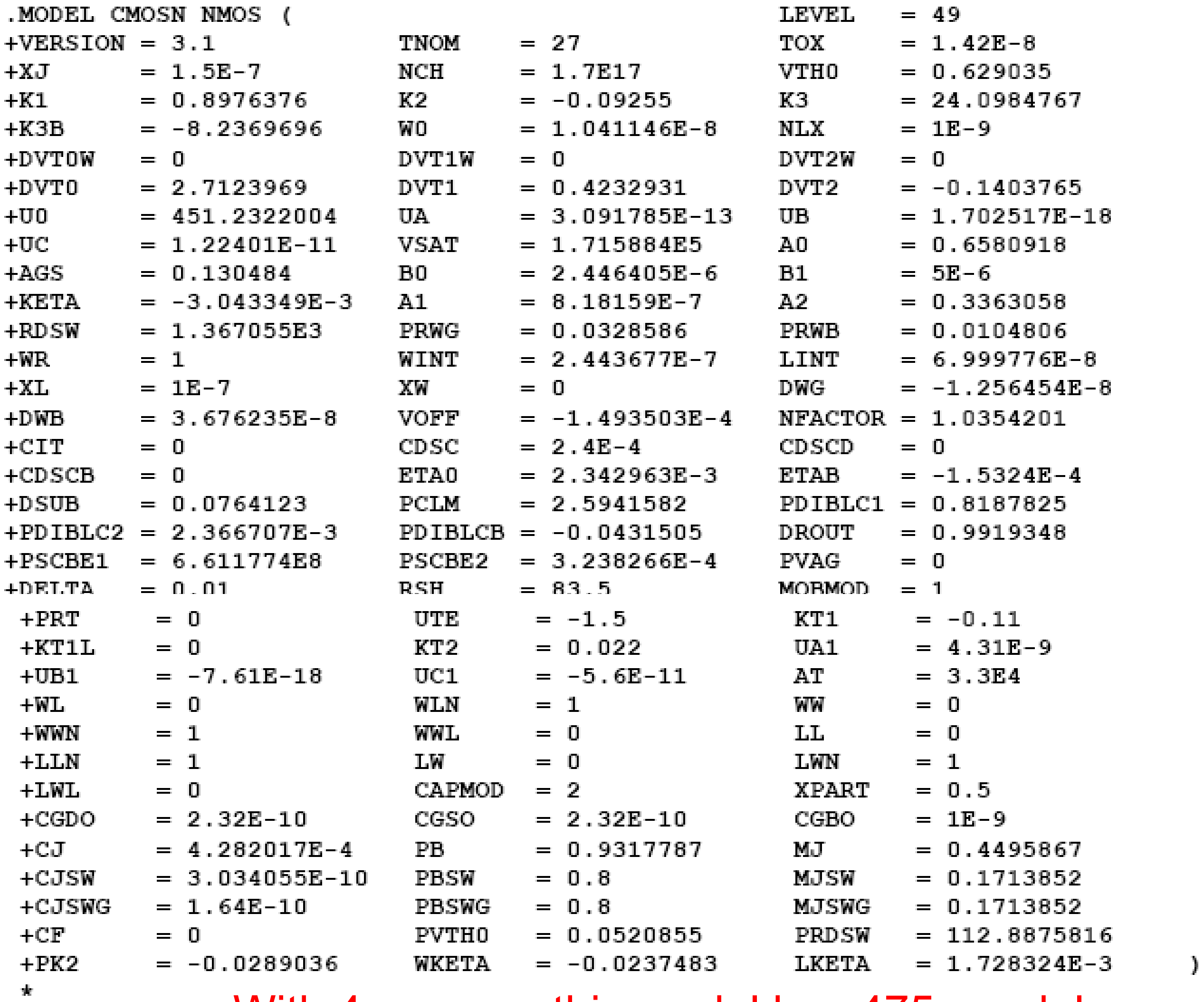
With 32 bins, this model has 3040 model parameters !

BSIM Corner Models

- Often 4 corners in addition to nominal TT, FF, FS, SF, and SS

- five different BSIM models !

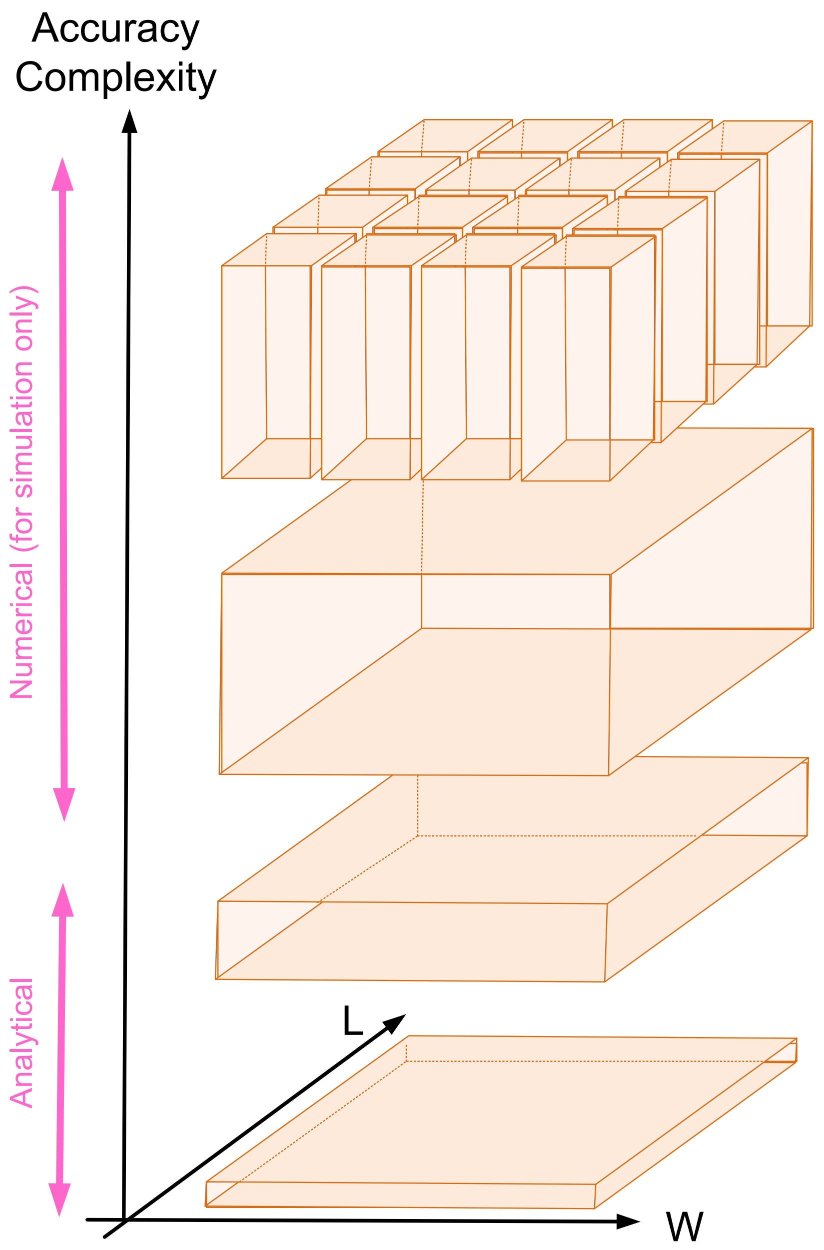
TT: typical-typical
FF: fast n, fast p
FS: fast n, slow p
SF: slow n, fast p
SS: slow n, slow p



```
.MODEL CMOSN NMOS (                                LEVEL   = 49
+VERSION = 3.1           TNOM    = 27              TOX     = 1.42E-8
+XJ      = 1.5E-7        NCH     = 1.7E17          VTH0    = 0.629035
+K1      = 0.8976376     K2      = -0.09255        K3      = 24.0984767
+K3B     = -8.2369696    W0      = 1.041146E-8     NLX     = 1E-9
+DVT0W   = 0             DVT1W   = 0               DVT2W   = 0
+DVT0    = 2.7123969     DVT1    = 0.4232931       DVT2    = -0.1403765
+U0      = 451.2322004   UA      = 3.091785E-13    UB      = 1.702517E-18
+UC      = 1.22401E-11   VSAT    = 1.715884E5      A0      = 0.6580918
+AGS     = 0.130484      B0      = 2.446405E-6     B1      = 5E-6
+KETA    = -3.043349E-3  A1      = 8.18159E-7      A2      = 0.3363058
+RDSW    = 1.367055E3    PRWG    = 0.0328586       PRWB    = 0.0104806
+WR      = 1             WINT    = 2.443677E-7     LINT    = 6.999776E-8
+XL      = 1E-7          XW      = 0               DWG     = -1.256454E-8
+DWB     = 3.676235E-8   VOFF    = -1.493503E-4    NFACTOR = 1.0354201
+CIT     = 0             CDSC    = 2.4E-4          CDSCD   = 0
+CDSCB   = 0             ETA0    = 2.342963E-3     ETAB    = -1.5324E-4
+DSUB    = 0.0764123     PCLM    = 2.5941582       PDIBLC1 = 0.8187825
+PDIBLC2 = 2.366707E-3   PDIBLCB = -0.0431505      DROUT   = 0.9919348
+PSCBE1  = 6.611774E8    PSCBE2  = 3.238266E-4     PVAG    = 0
+DELTA   = 0.01          RSH     = 83.5            MOBMOD  = 1
 +PRT     = 0             UTE     = -1.5            KT1     = -0.11
 +KT1L    = 0             KT2     = 0.022           UA1     = 4.31E-9
 +UB1     = -7.61E-18     UC1     = -5.6E-11        AT      = 3.3E4
 +WL      = 0             WLN     = 1               WW      = 0
 +WWN     = 1             WWL     = 0               LL      = 0
 +LLN     = 1             LW      = 0               LWN     = 1
 +LWL     = 0             CAPMOD  = 2               XPART   = 0.5
 +CGDO    = 2.32E-10      CGSO    = 2.32E-10        CGBO    = 1E-9
 +CJ      = 4.282017E-4   PB      = 0.9317787       MJ      = 0.4495867
 +CJSW    = 3.034055E-10  PBSW    = 0.8             MJSW    = 0.1713852
 +CJSWG   = 1.64E-10      PBSWG   = 0.8             MJSWG   = 0.1713852
 +CF      = 0             PVTH0   = 0.0520855       PRDSW   = 112.8875816
 +PK2     = -0.0289036    WKETA   = -0.0237483      LKETA   = 1.728324E-3     )
 *
```

With 4 corners, this model has 475 model parameters !

Hierarchical Model Comparisons



BSIM Binning Models

Number of Model Parameters — **Approx 3000 (for 30 bins)**

BSIM Models

Number of Model Parameters — **Approx 100**

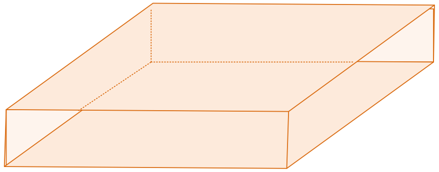
Square-Law Models

Number of Model Parameters — **3 to 6**

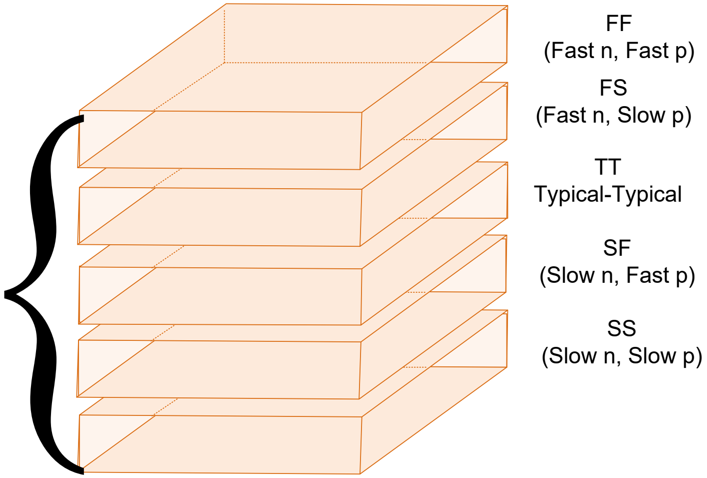
Switch-Level Models

Number of Model Parameters — **0 to 2**

Corner Models



Basic Model



Corner Model

Applicable at any level in model hierarchy (same model, different parameters)

Often 4 corners (FF, FS, SF, SS) used but sometimes many more

Designers must provide enough robustness so good yield at all corners

Stay Safe and Stay Healthy !

End of Lecture 17